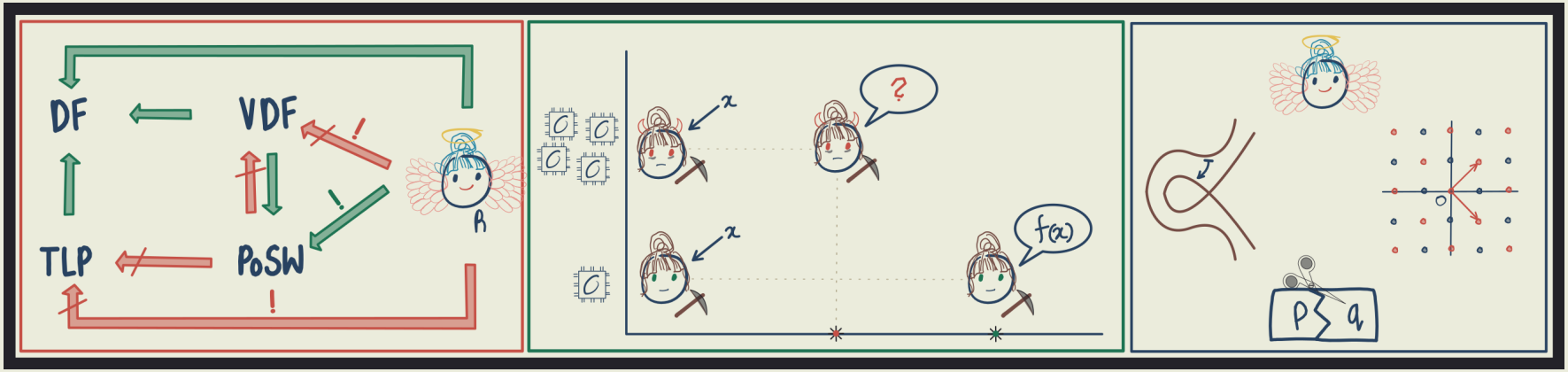

Timed Cryptography: Attacks and Separations

Chethan Kamath, IIT Bombay & Trust Lab



Based on joint works with Hamza Abusalah, Nivesh Aggarwal, Karen Azari, Erkan Tairi & Max von Consbruch

aRtCS Workshop, 09-13/Nov/26

Plan for Today's Talk



Timed Cryptography



Current Landscape



Breaking Random-Oracle Based VDF

Plan for Today's Talk



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What is a Timed Primitive?..



Primitives that deal in “wall-time”

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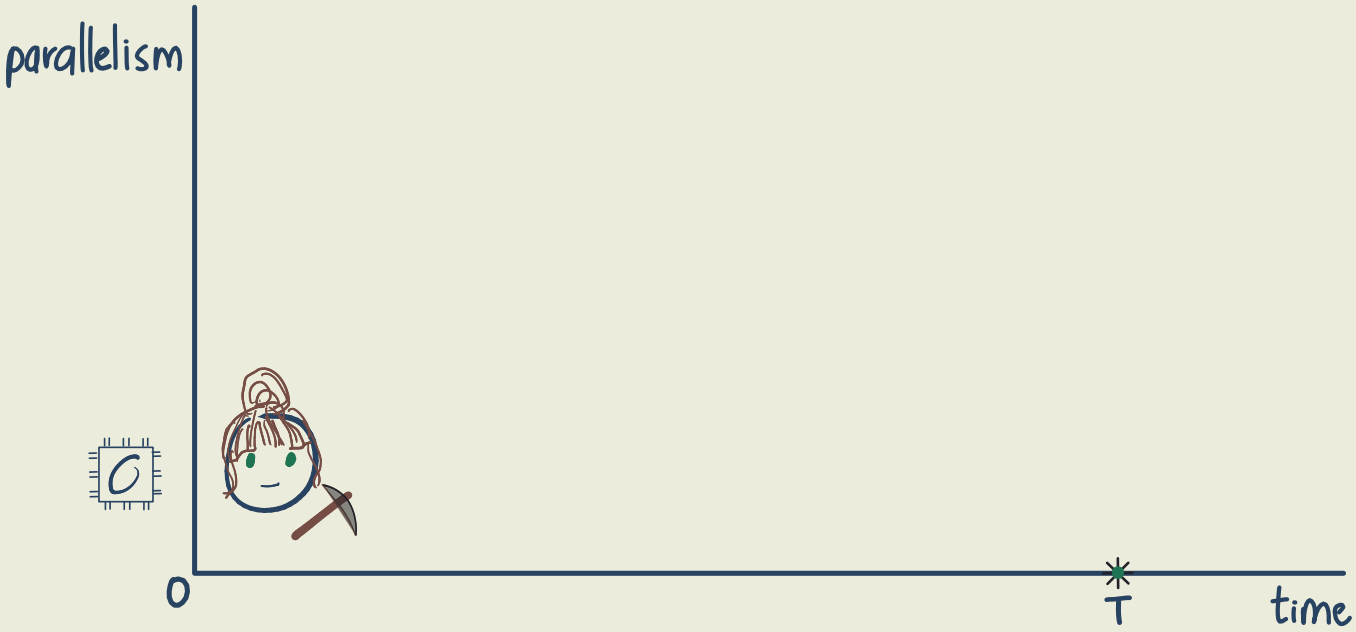

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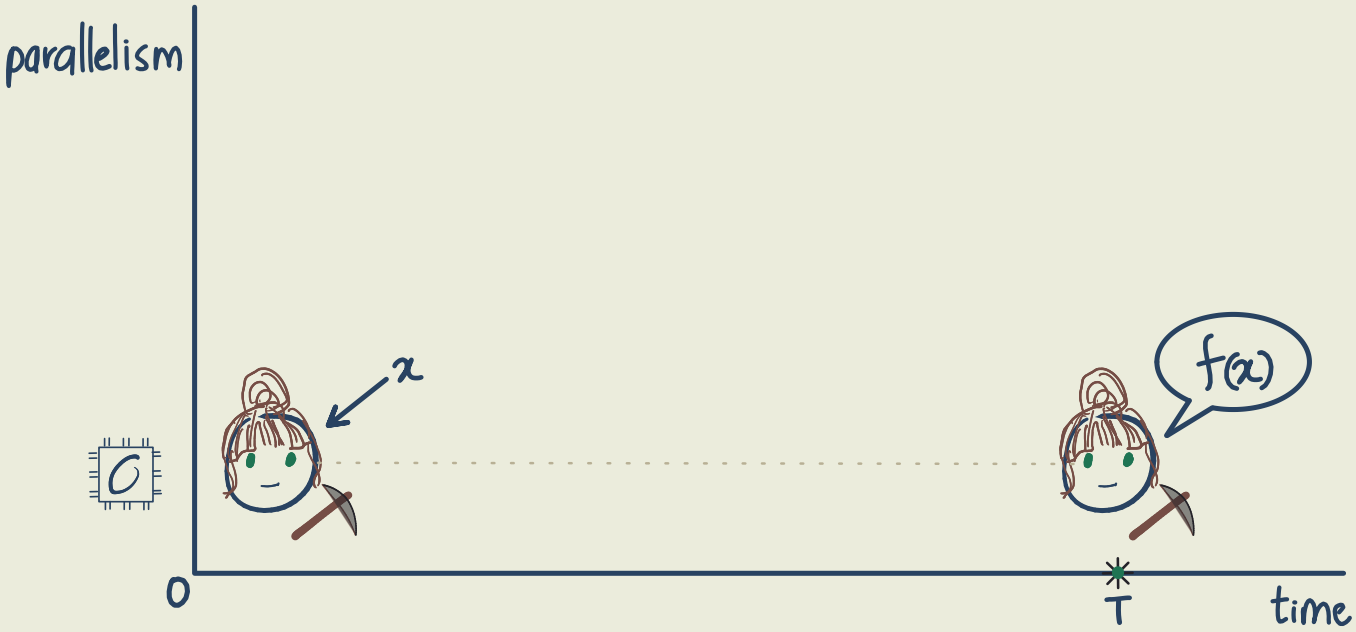

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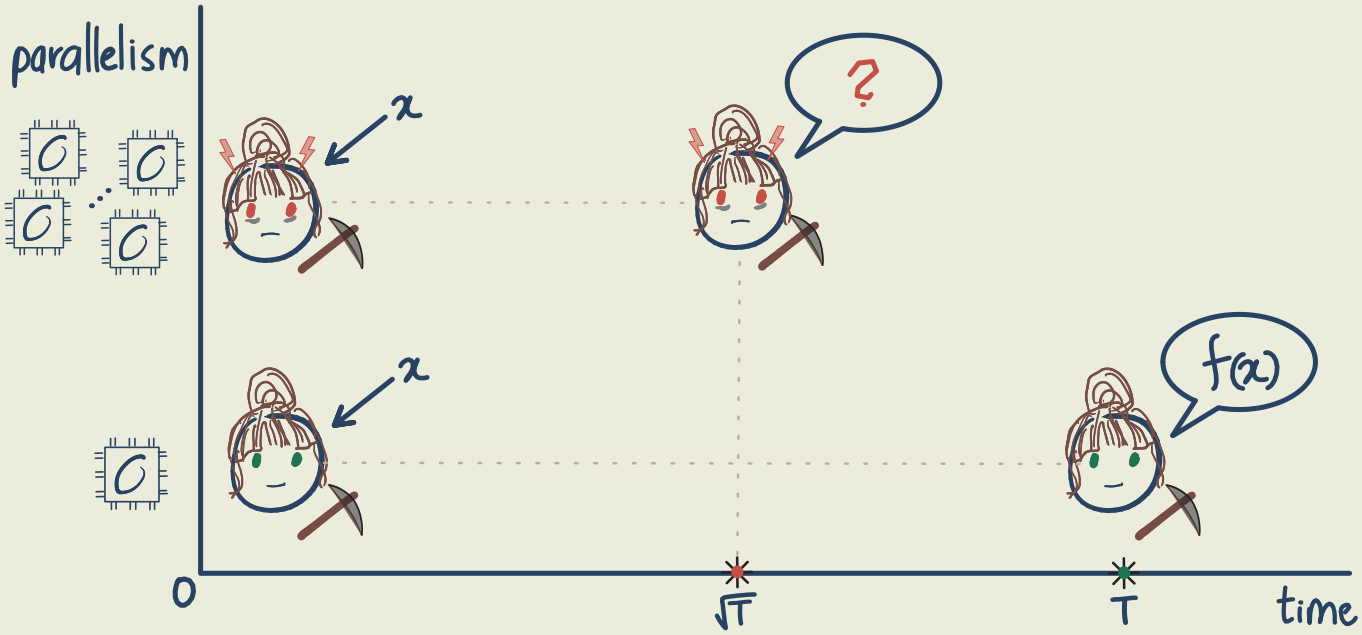

- Simplest example: delay function (DF) $f : \mathcal{X} \to \mathcal{Y}$
  - i) Can be evaluated in time T; but
  - ii) Cannot be evaluated in time $\ll T$ even given parallelism

Alternatively: primitives secure only for a predetermined wall-time T

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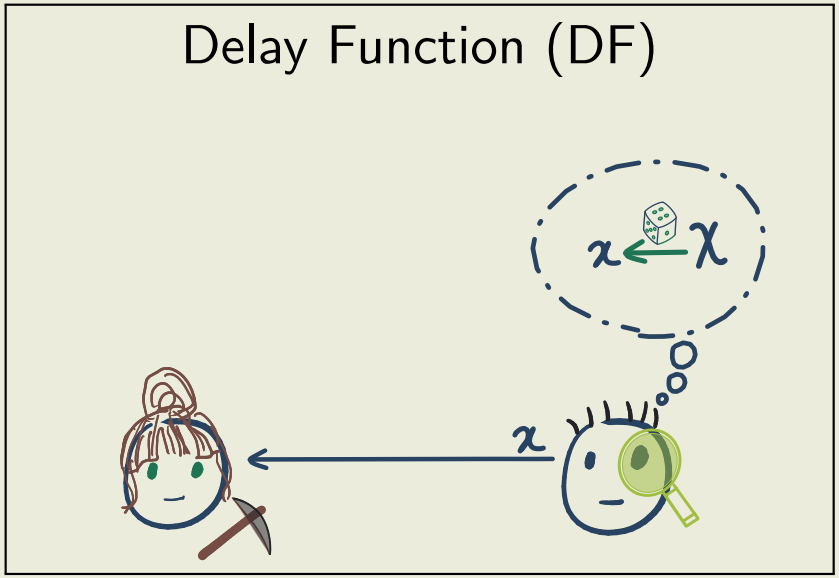

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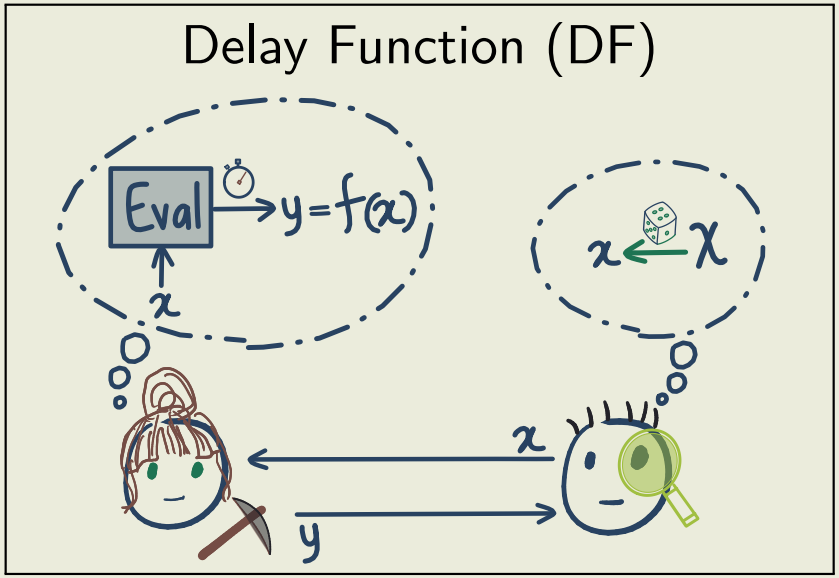

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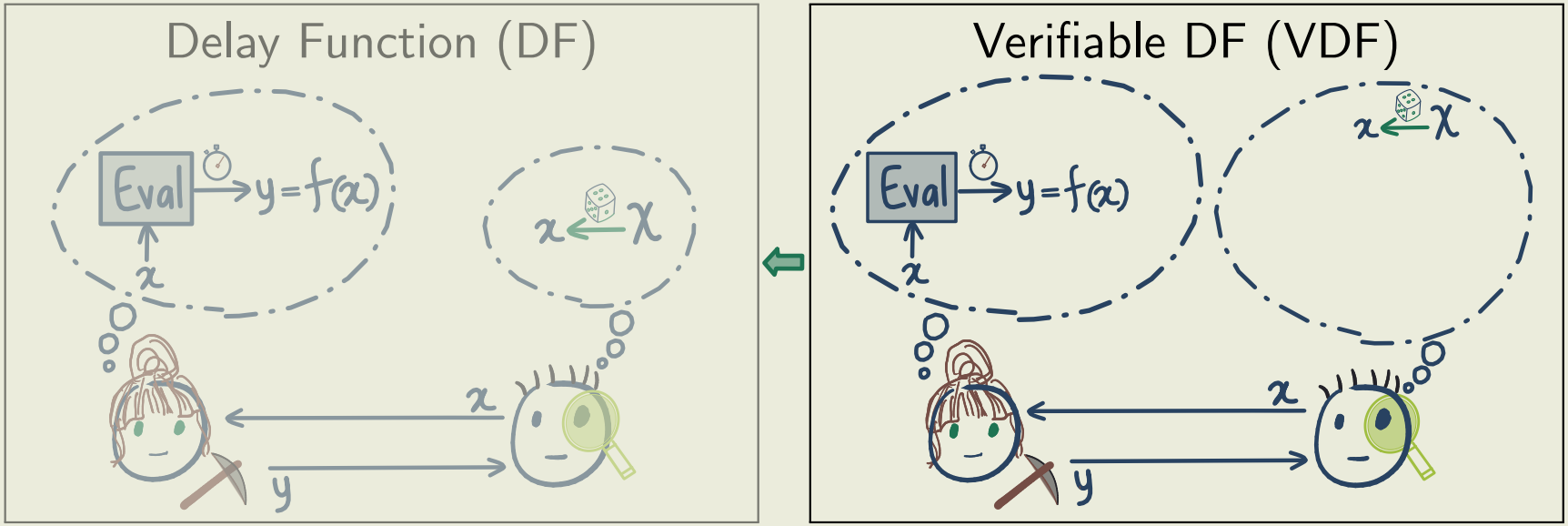

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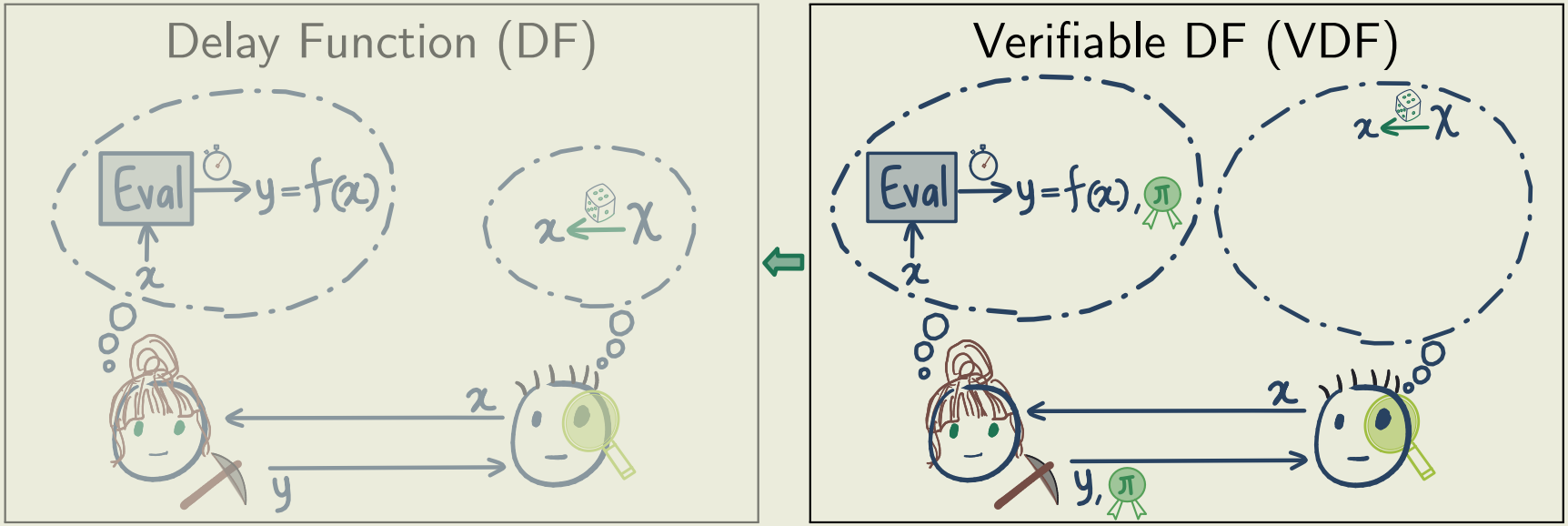

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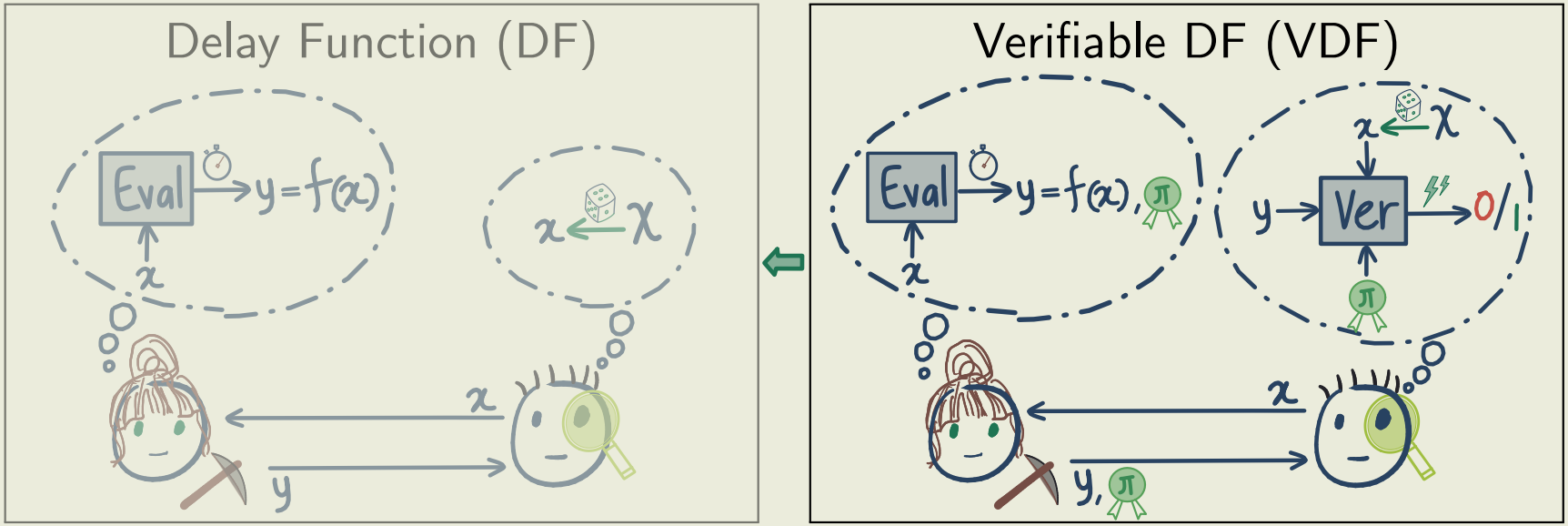

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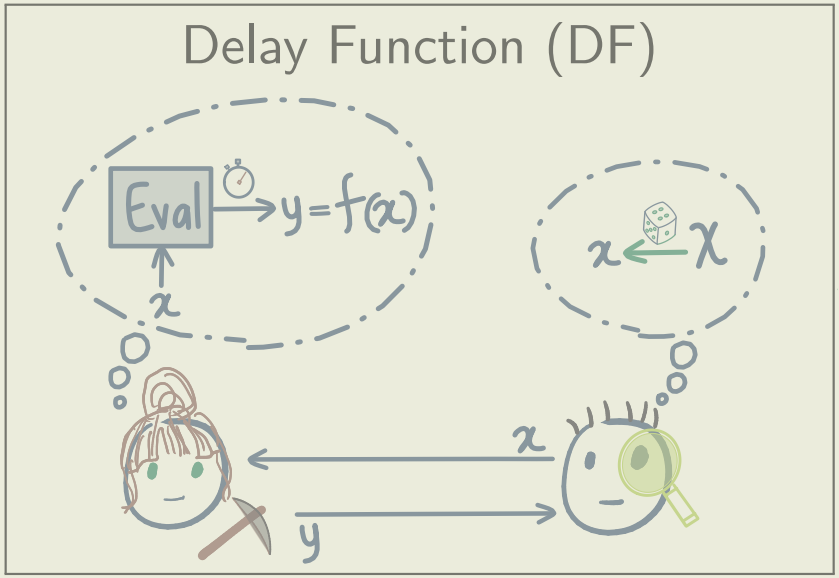

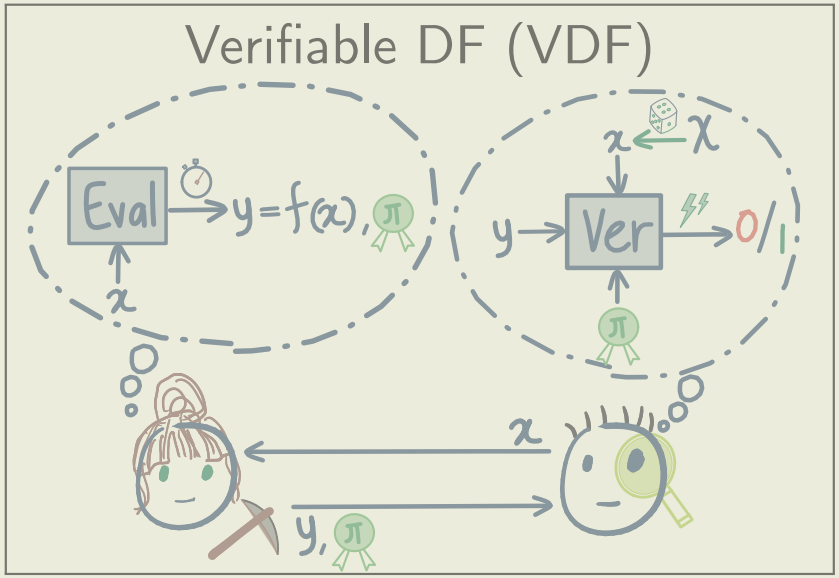

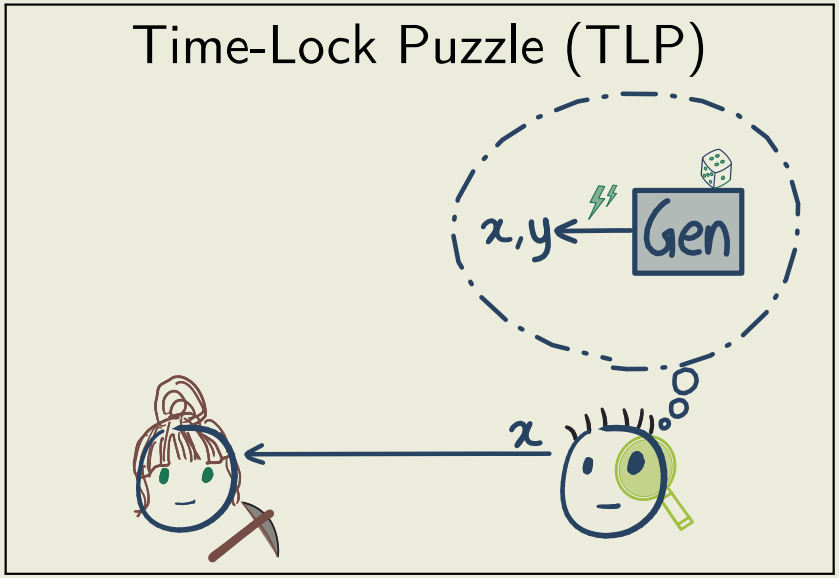

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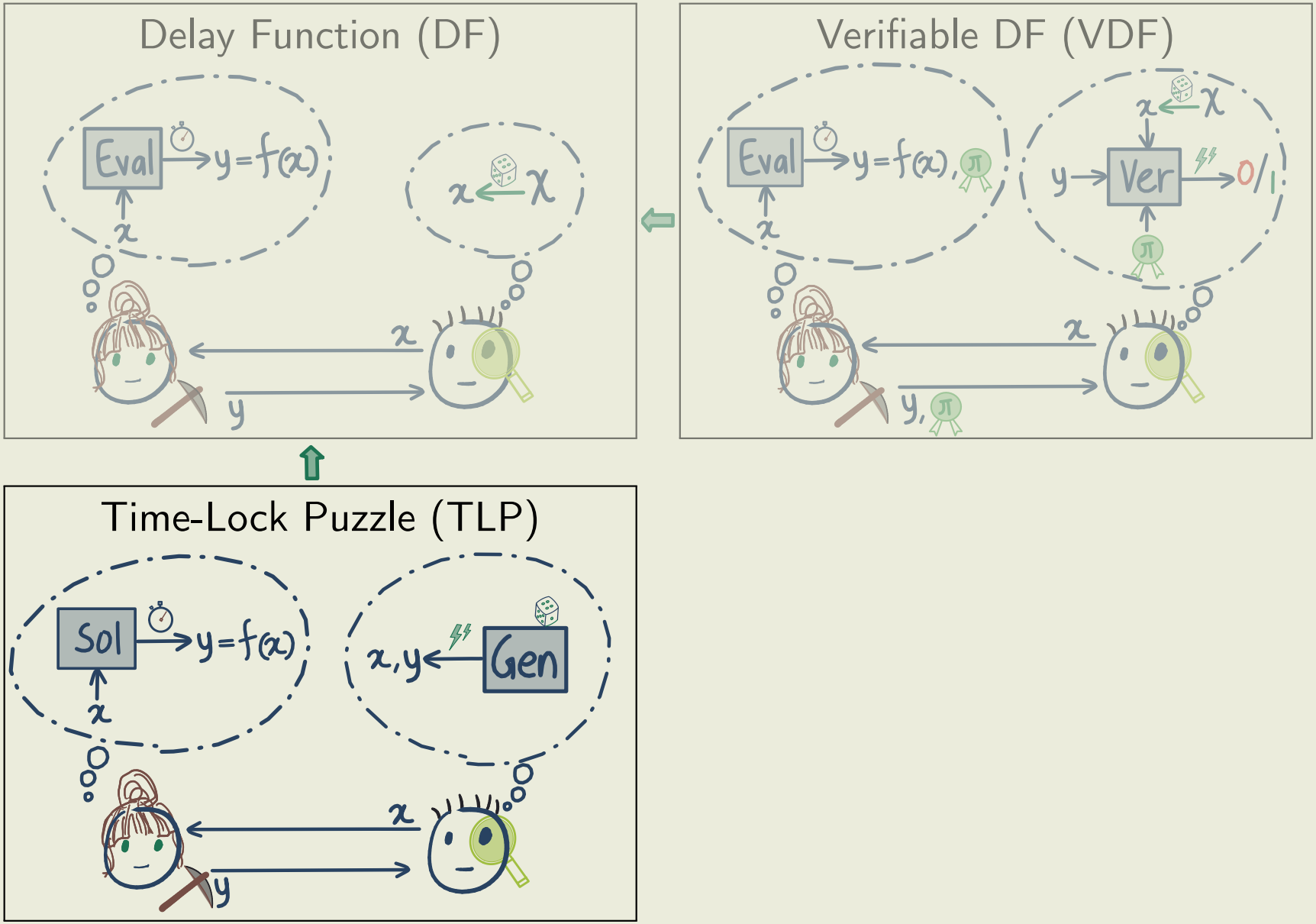

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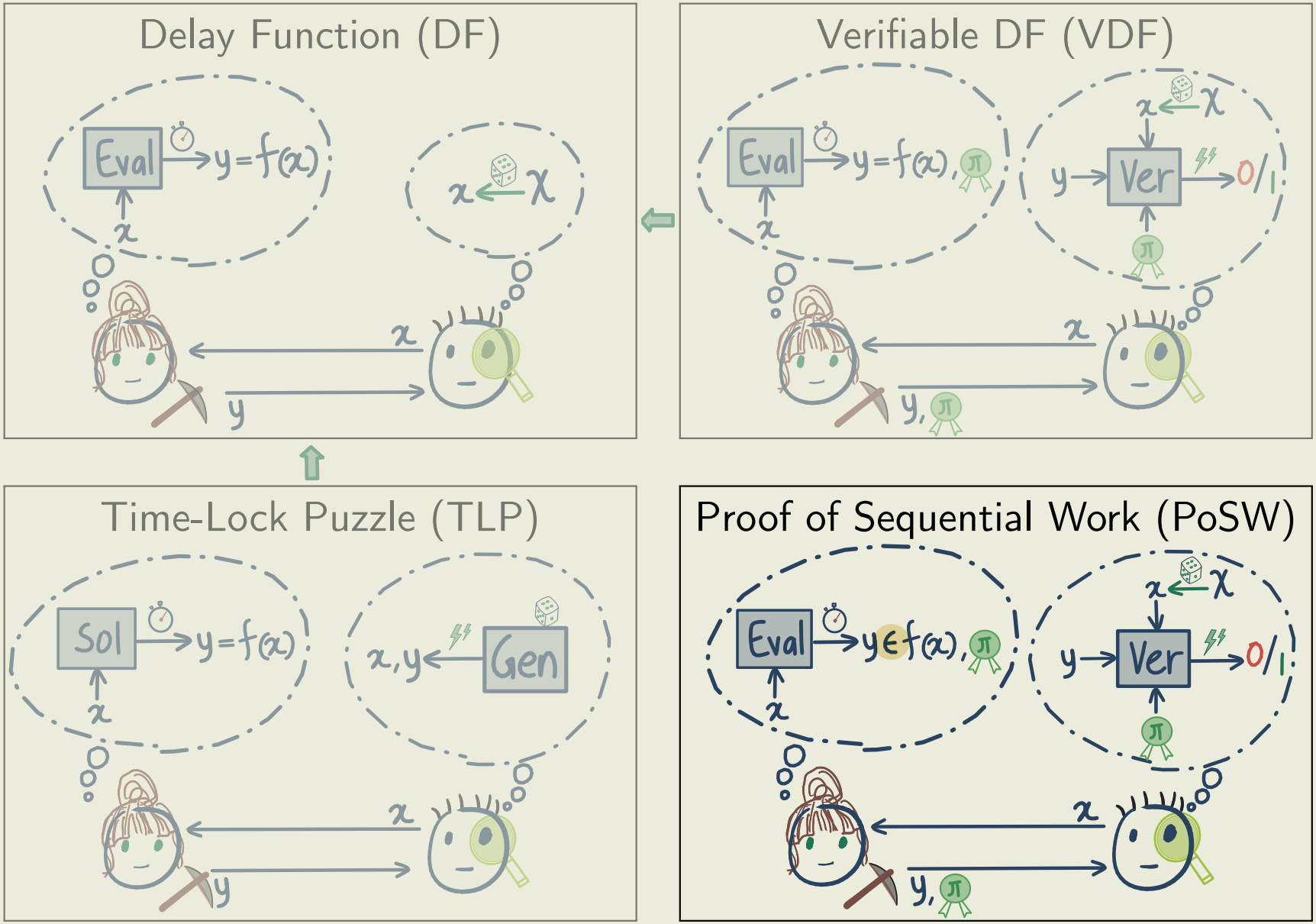

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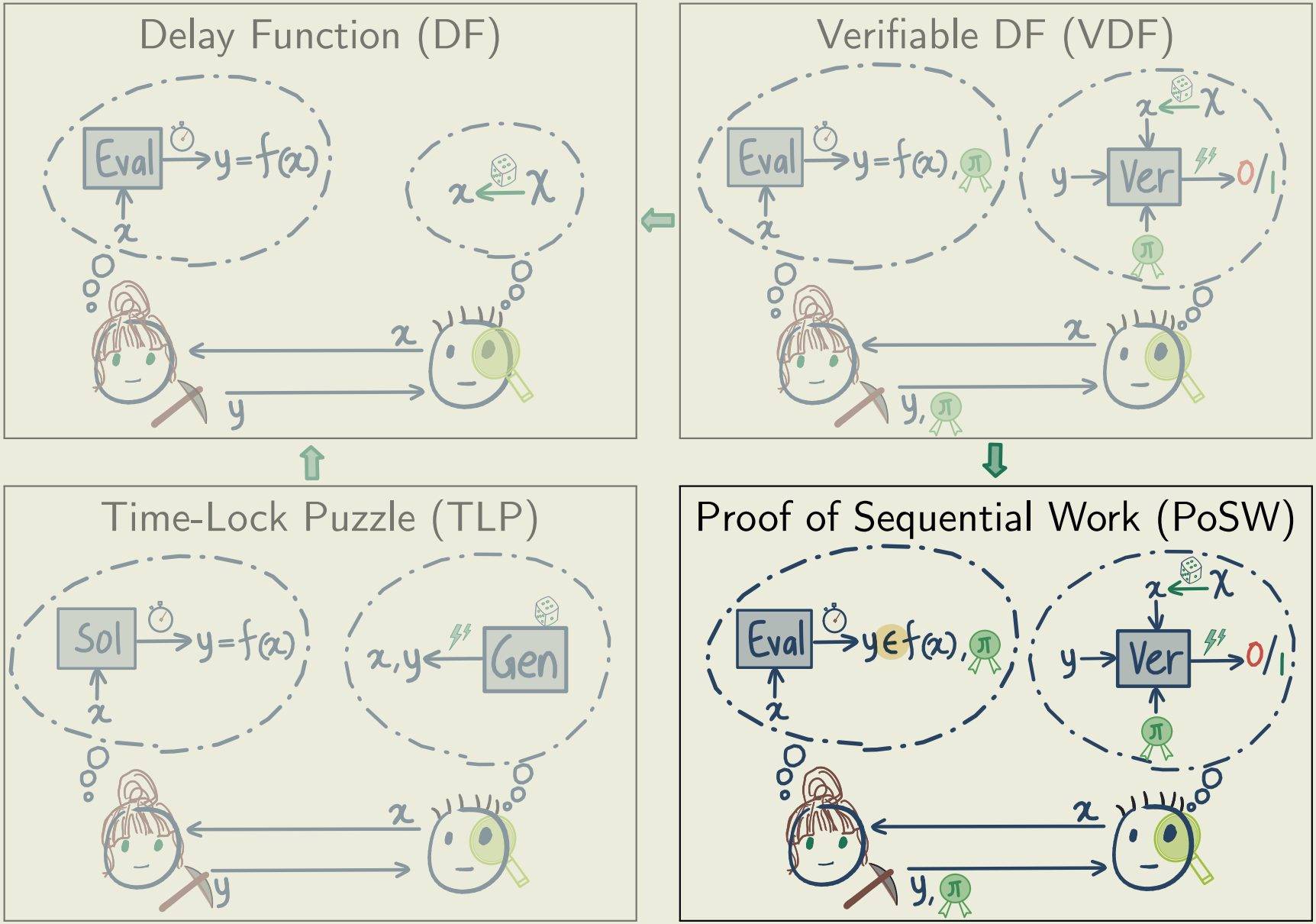

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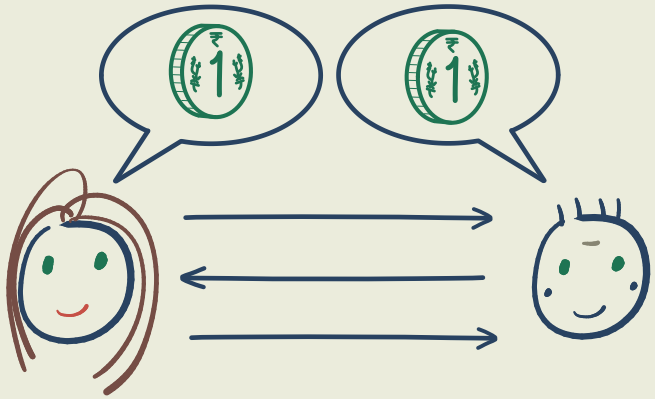

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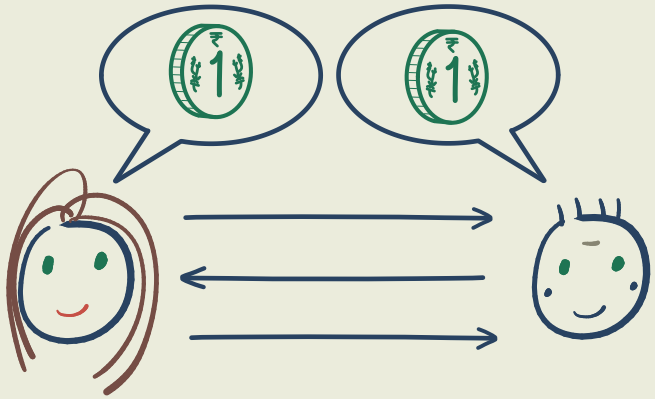

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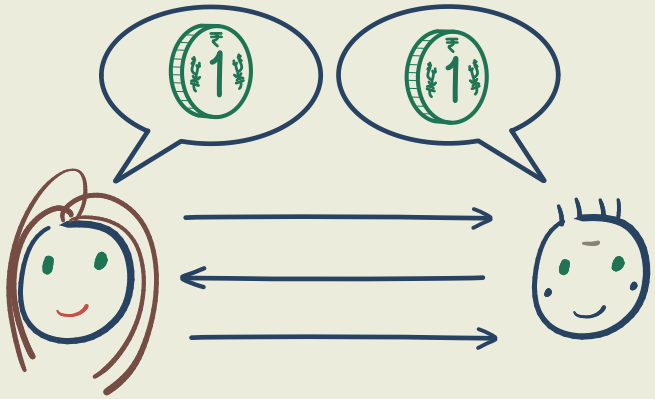

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2. Randomness beacons[LW17]
3. Benchmarking hardware[CLSY93]
4. Deniability, via short-lived signatures[ABC22]

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VDF[Pie19]: to get public verifiability for $f_{N,T}$,

i) Design public-coin interactive proof Π for claim "$y = f_{N,T}(x)$"
ii) Make non-interactive using Fiat-Shamir, e.g., using random oracle

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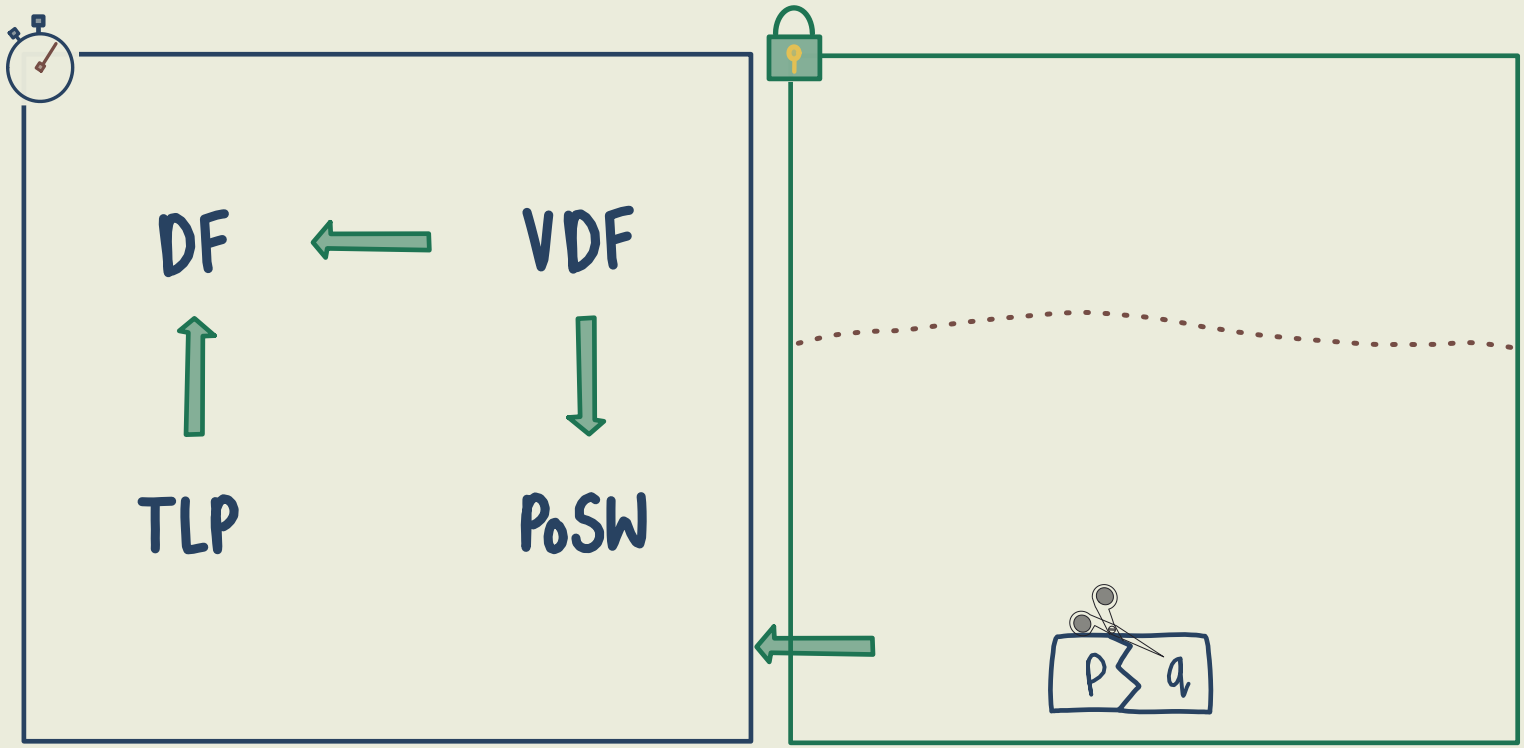

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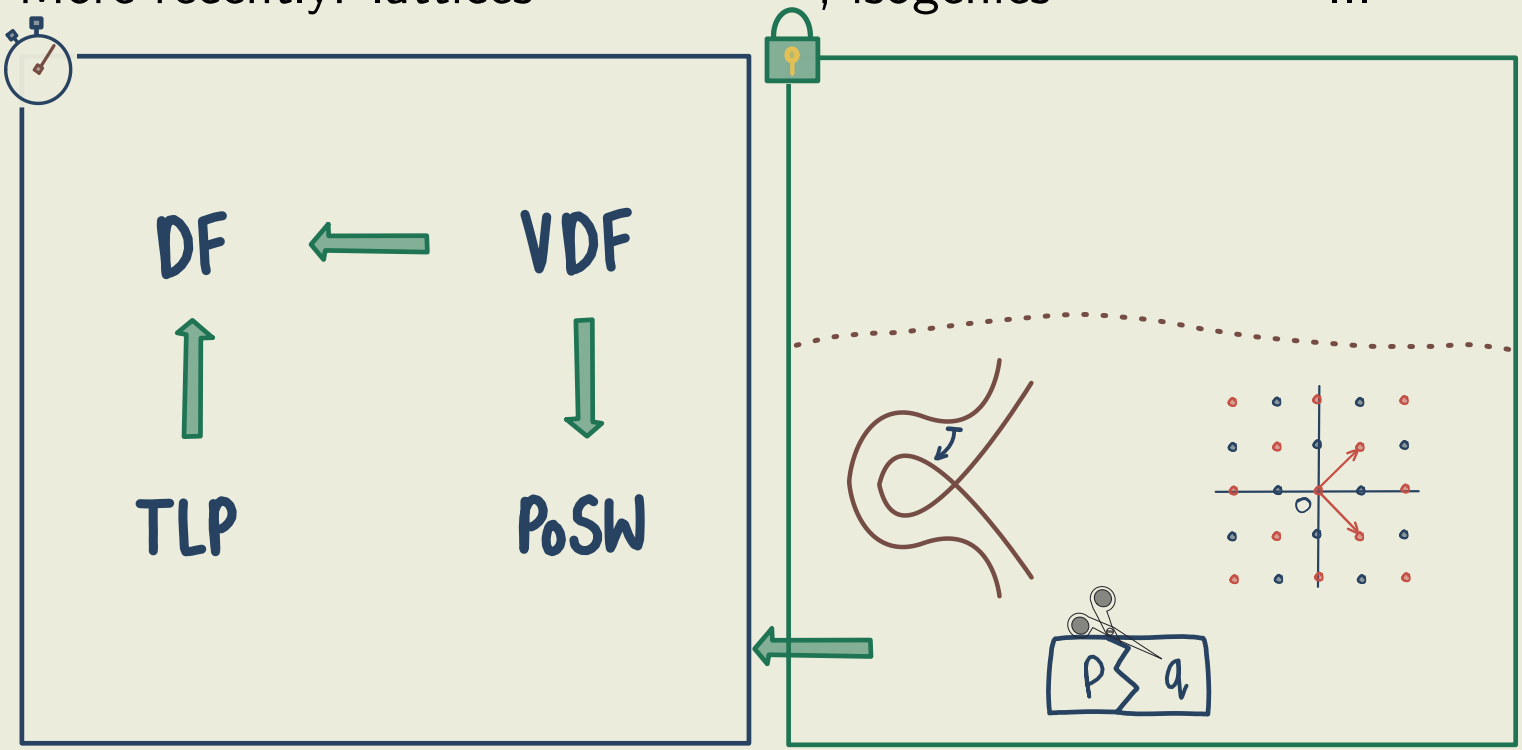

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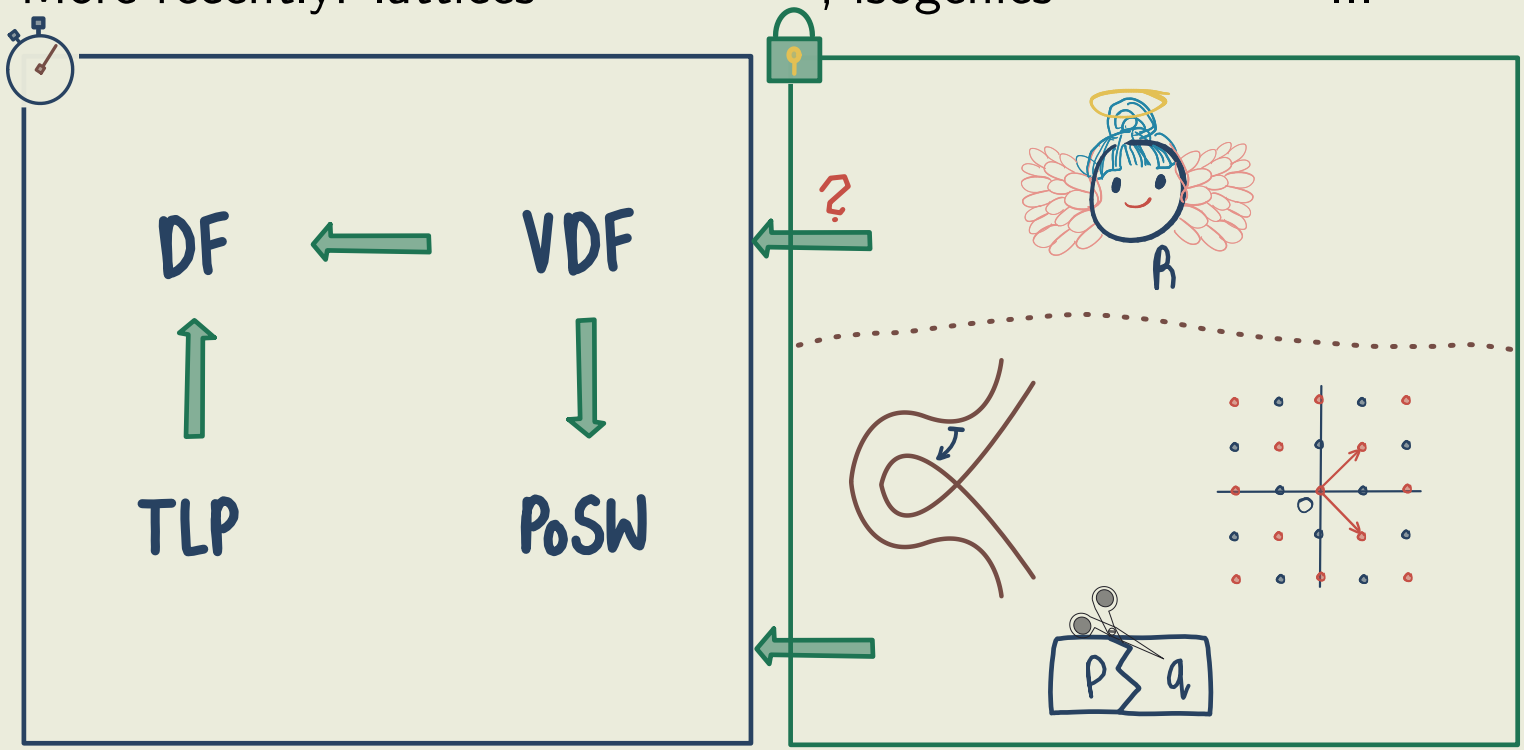

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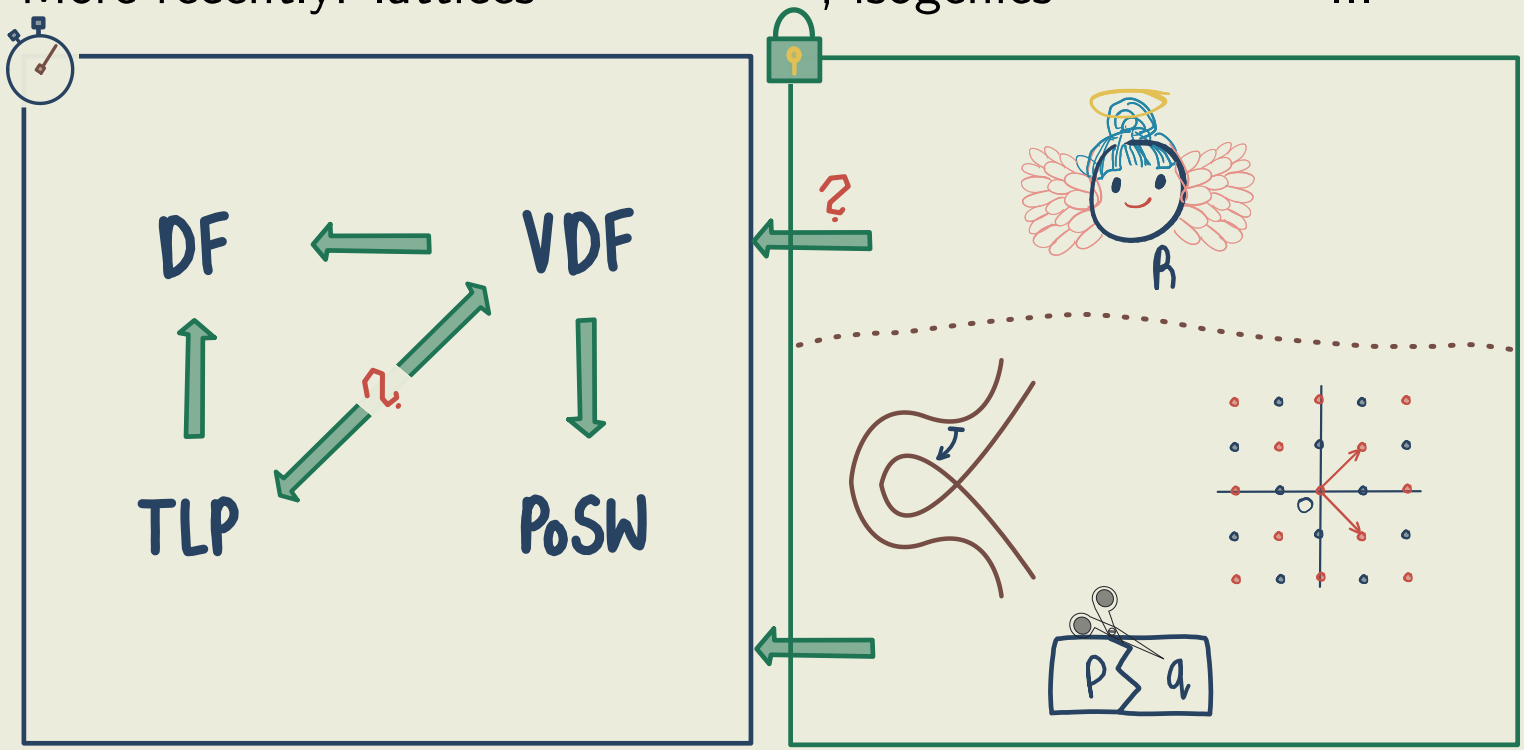
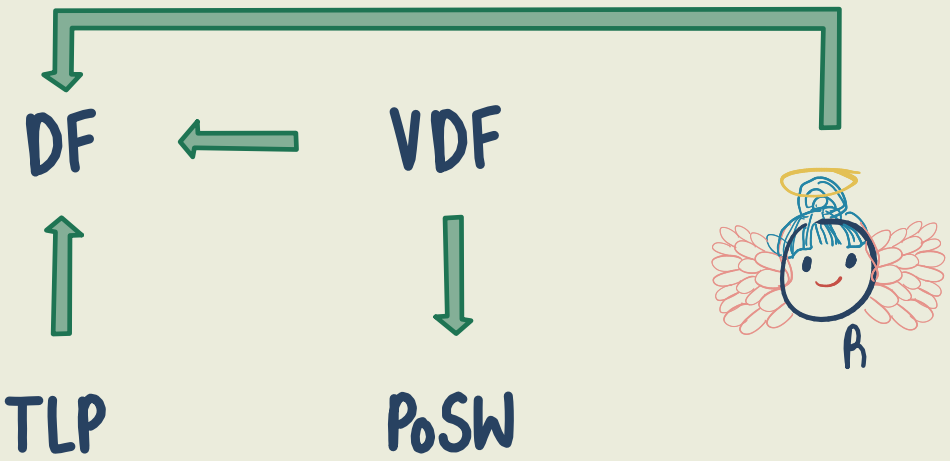


3. What about unstructured primitives, say (only) a random oracle?
4. What about *other* timed primitives?
   i) Can we construct VDF from TLP?
   ii) And vice versa?

Timed Primitives from Random Oracle (RO)



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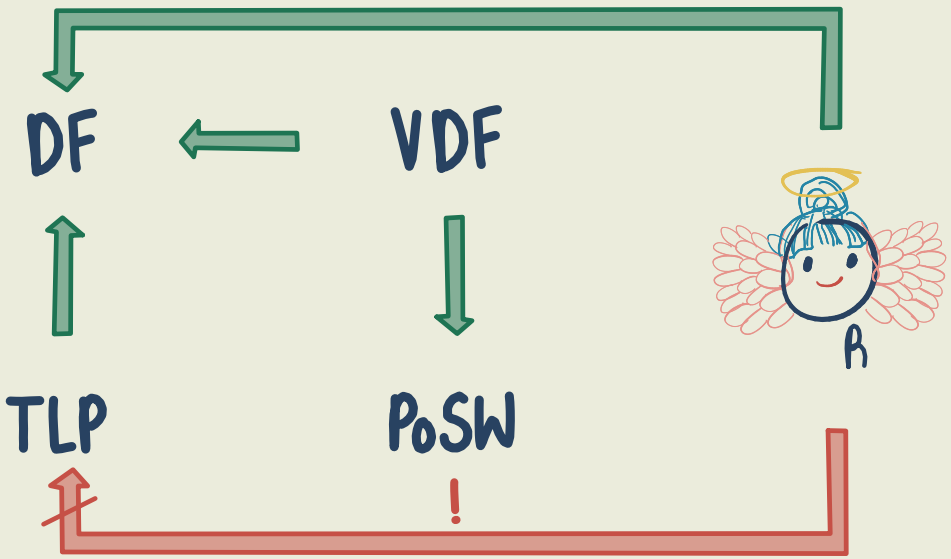

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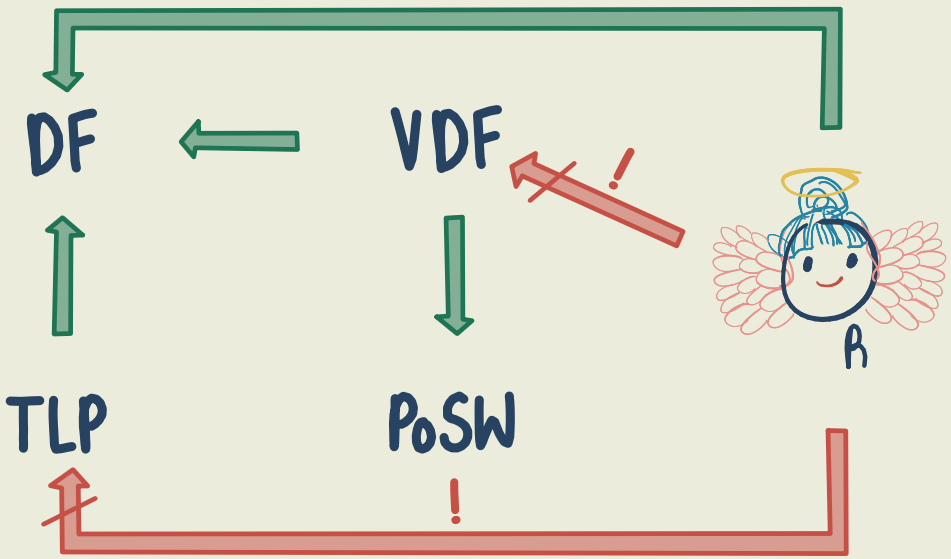

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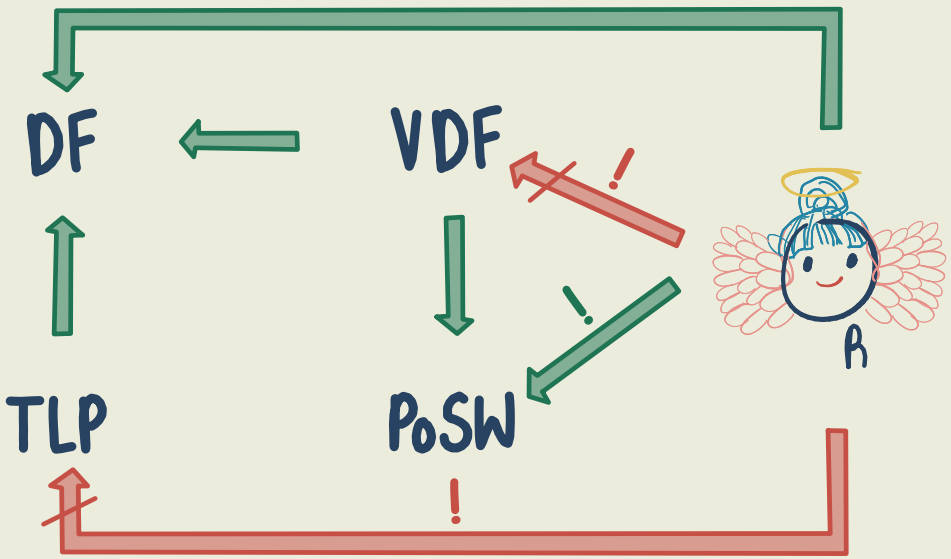

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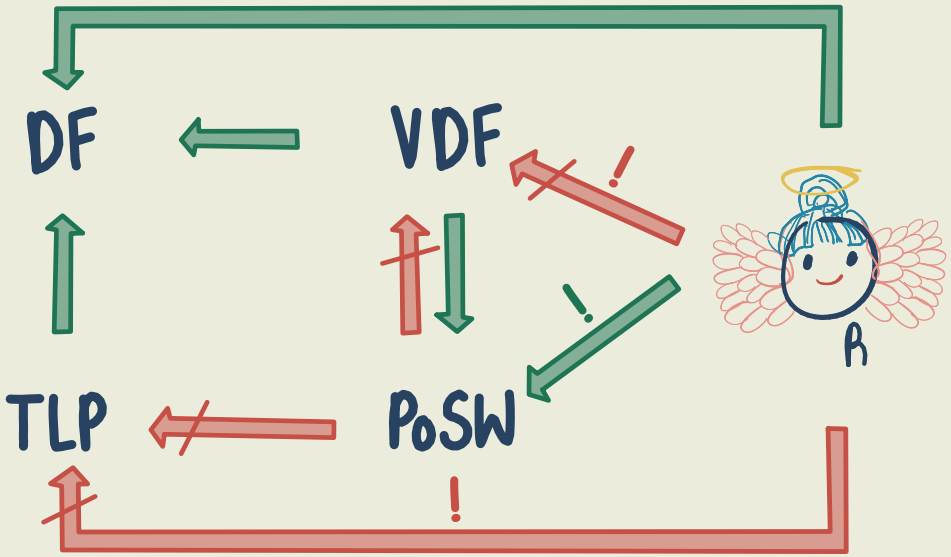

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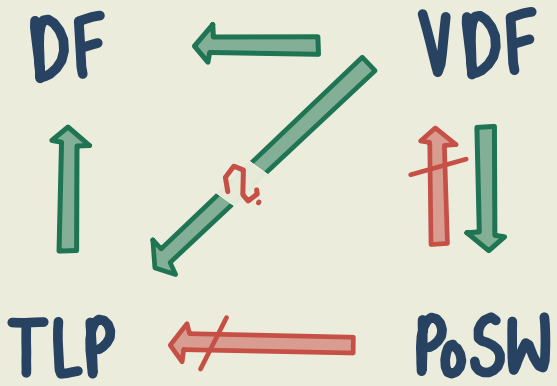

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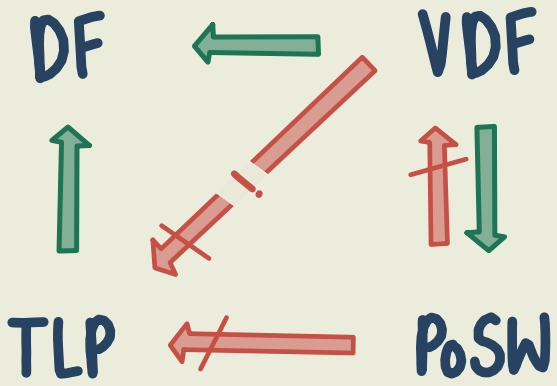

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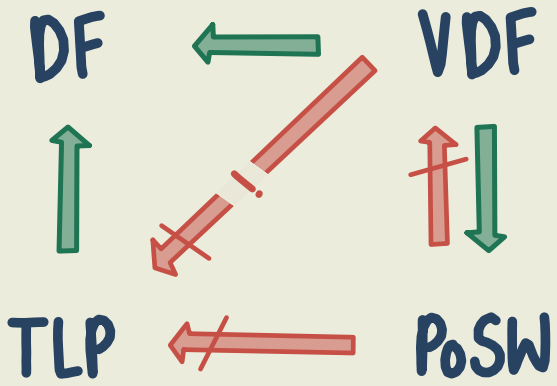

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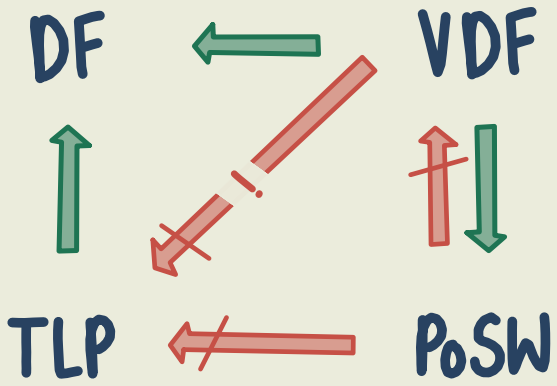

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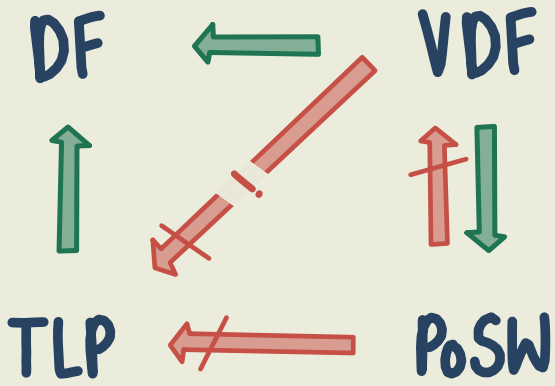

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     i) The use of V_R in the TLP construction can be "simulated"
     ii) Hence TLP can be constructed only from R (not possible[MMV11])

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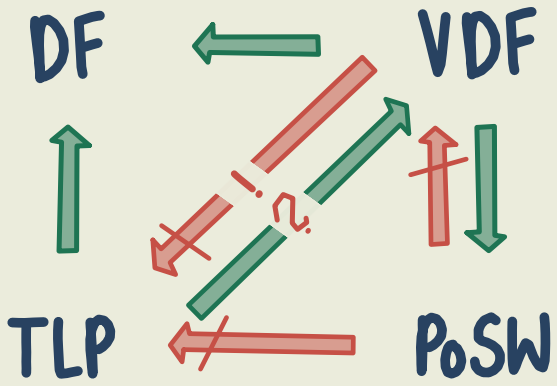

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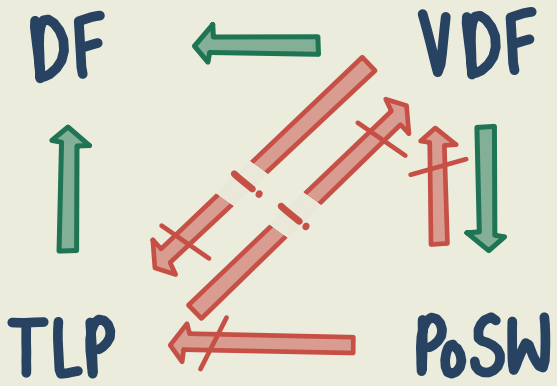

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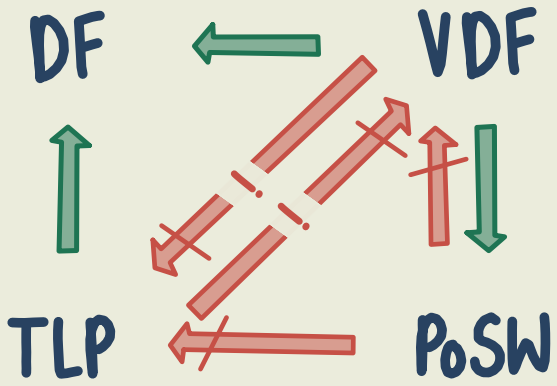

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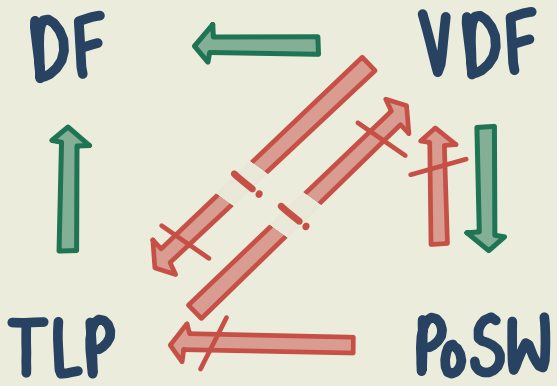

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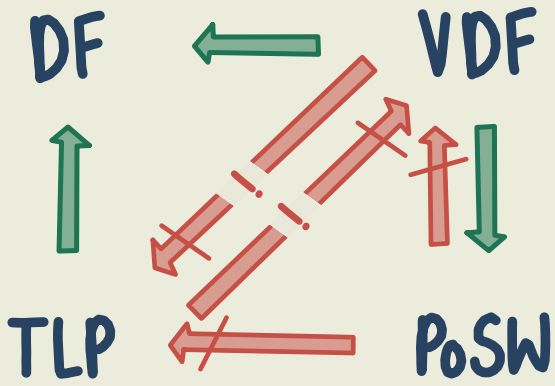
   + Why TLP trivially exists in W_2?

     i) Generating a puzzle with solution is free given S_P: sample solution x and set $y := S_P(x, T)$ as the puzzle

     ii) Solve puzzle using T sequential calls to P^{-1}

Timed Primitives from Other Timed Primitives..



2. VDF $\Leftarrow$ TLP? Not possible: *there exists a world* $W_2 := (P, P^{-1}, S_P)$ *in which a TLP exists but VDFs don't.*[AAA+26]

   - P is a random permutation, P^{-1} is its inverse and S_P is an auxiliary "shortcut" oracle such that that $S_P(x, t) := P^t(x)$

   + Why TLP trivially exists in W_2?
     i) Generating a puzzle with solution is free given S_P: sample solution x and set $y := S_P(x, T)$ as the puzzle
     ii) Solve puzzle using T sequential calls to P^{-1}

   ! Why VDF cannot exist in W_2? Extend the ideas of why RO-based VDFs can be broken[AAA+26]

Plan for Today's Talk

Timed Cryptography

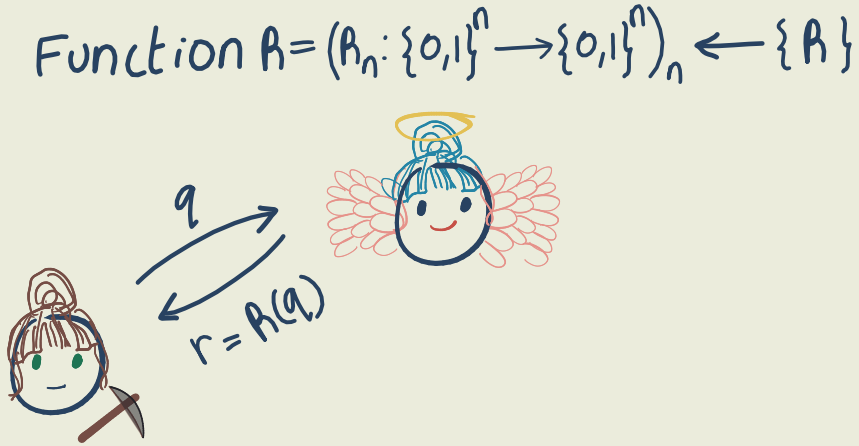


Current Landscape



Breaking Random-Oracle Based VDF

(Parallel) Random-Oracle Model



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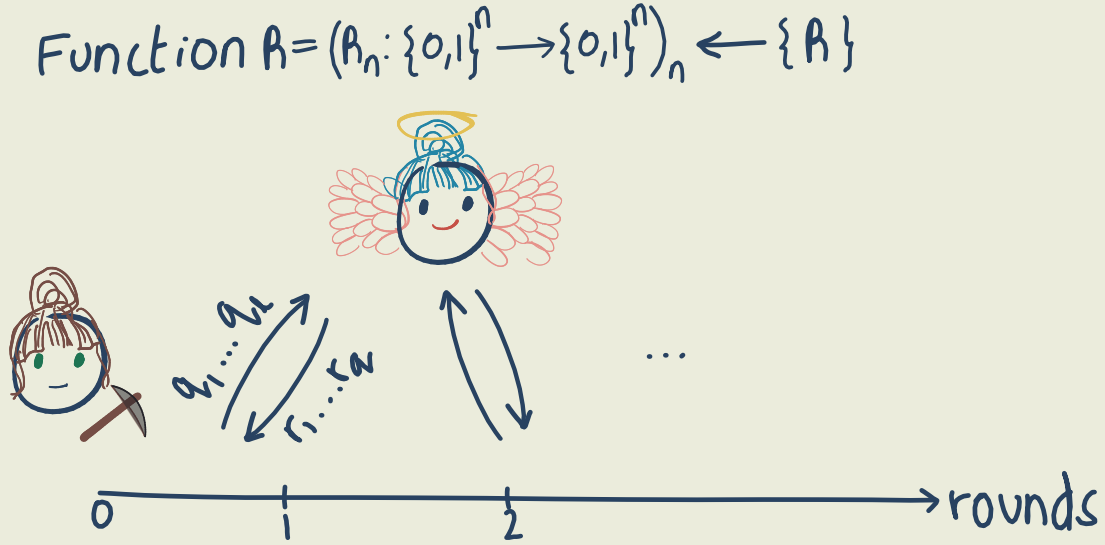

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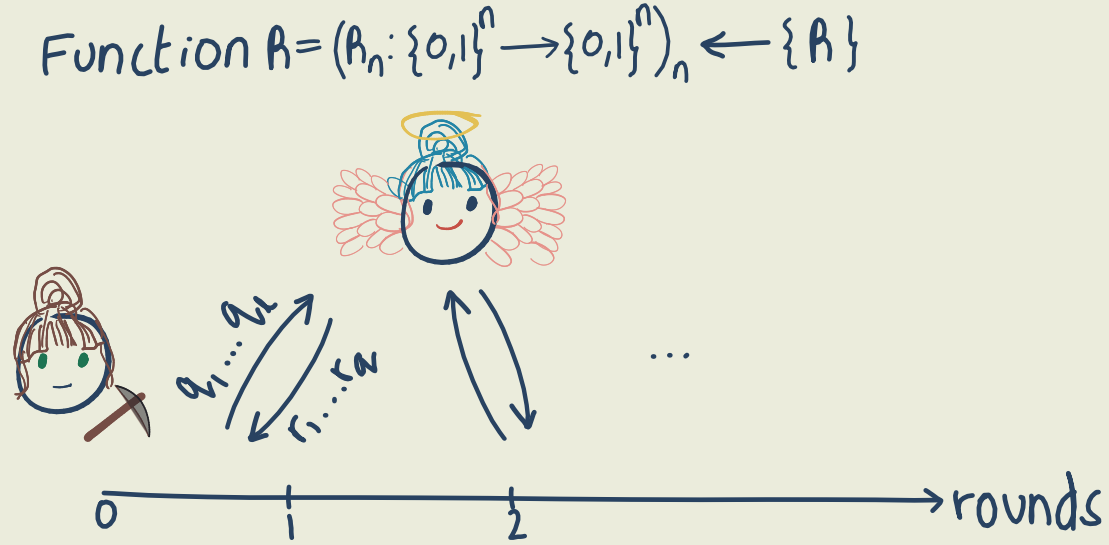

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₹ Complexity measures for an algorithm:

- ⏱ Round complexity: total number of *rounds* in which it queries R
- Σ Query complexity: total number of *queries* it makes to R

RO-Based VDF

is a triple of oracle algorithms $(\mathsf{Setup}^{\cdot}, \mathsf{Eval}^{\cdot}, \mathsf{Ver}^{\cdot})$ with following syntax:



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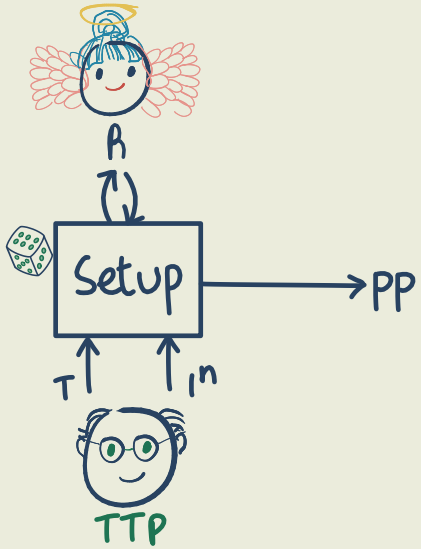

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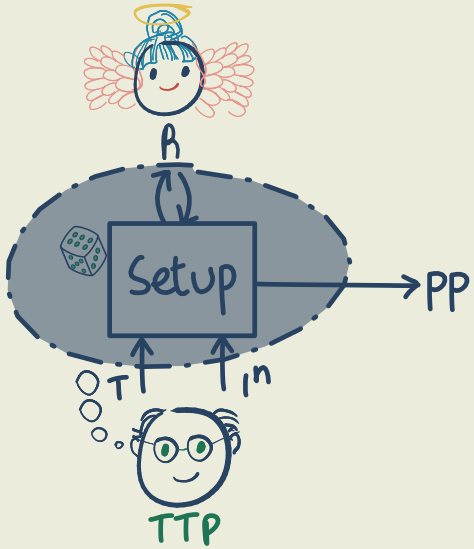

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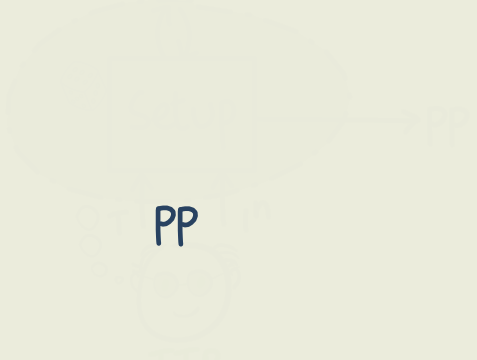

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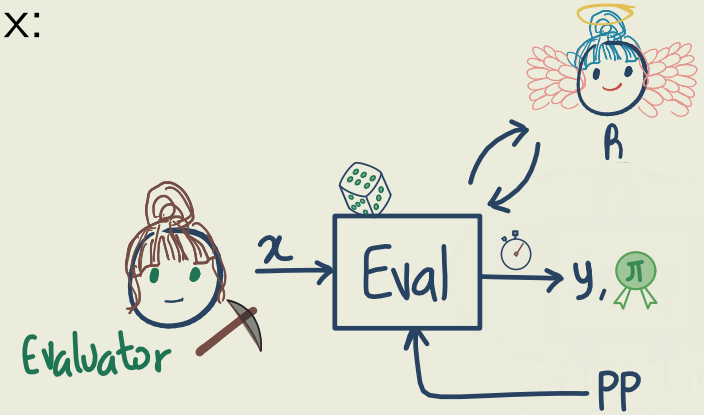

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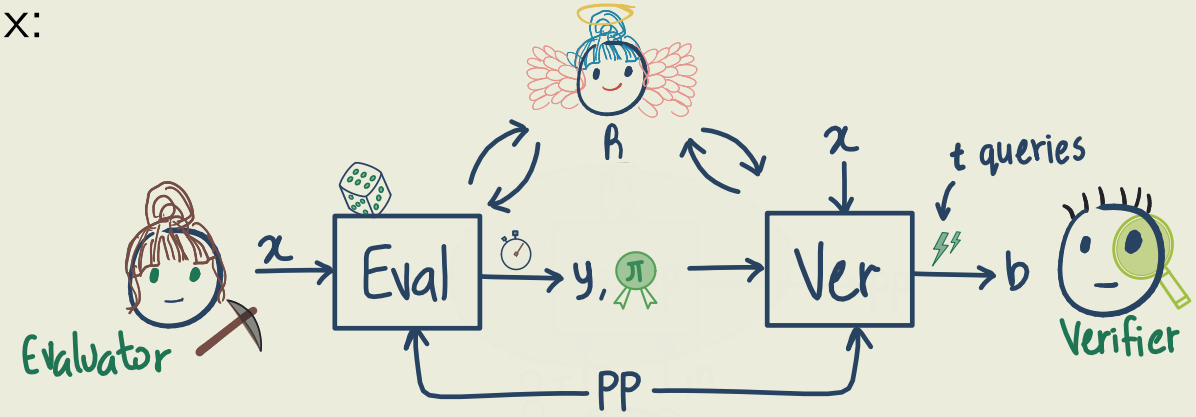

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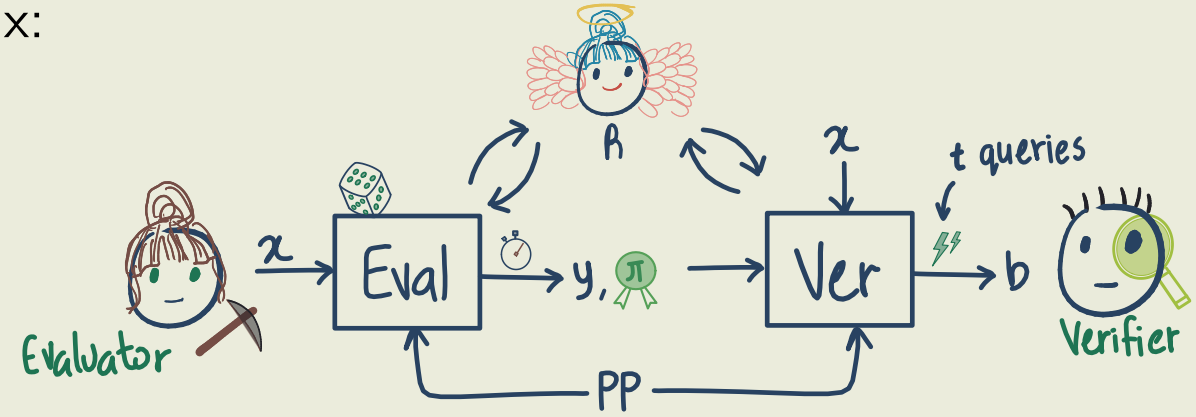

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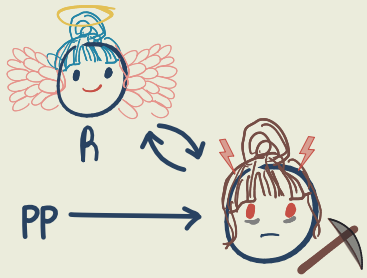

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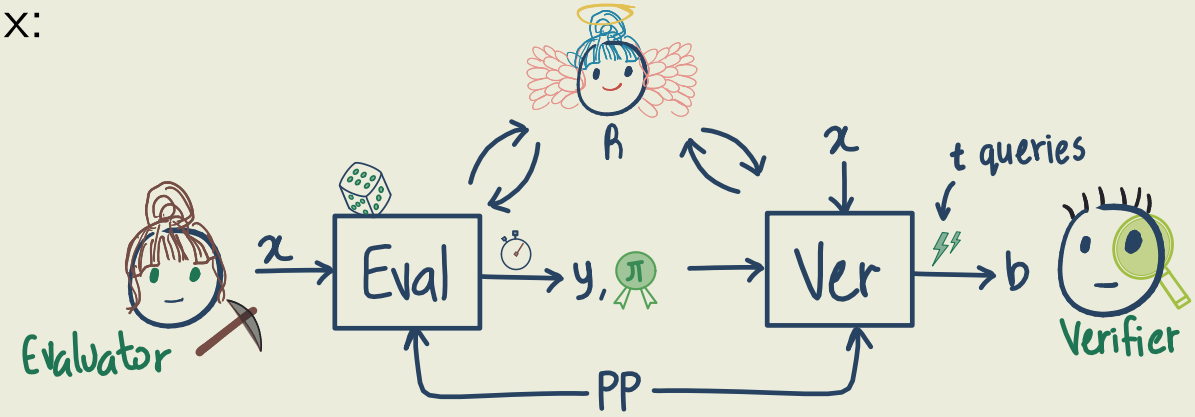

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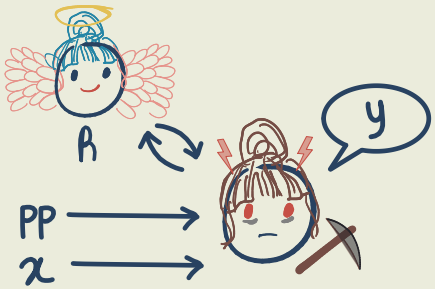

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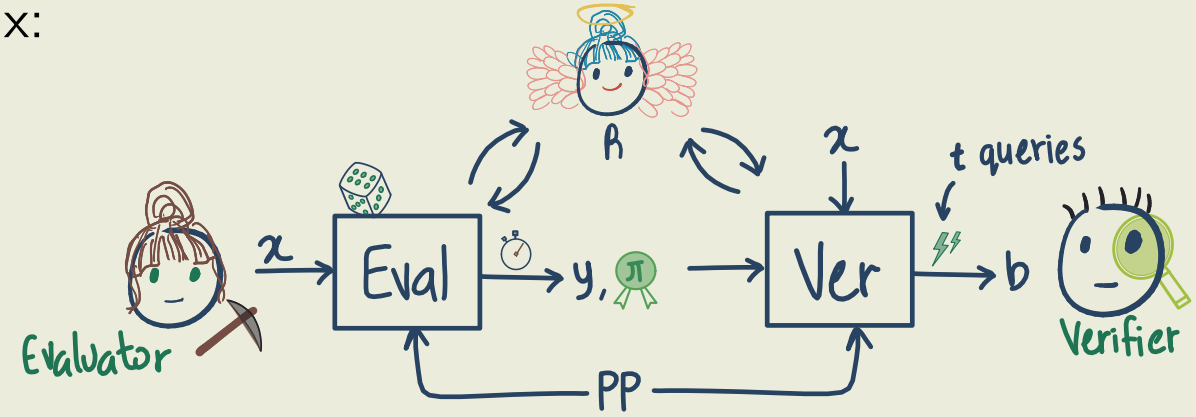

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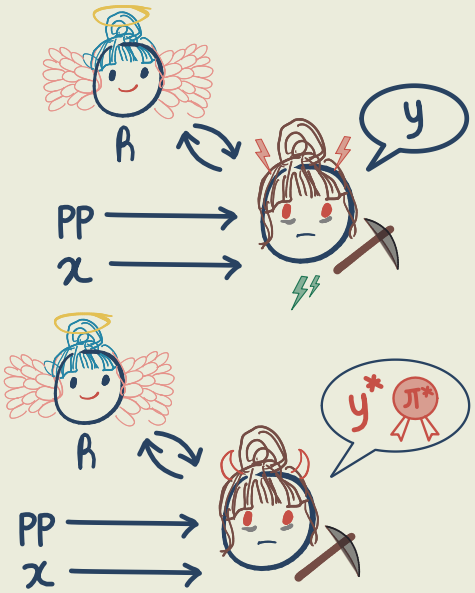

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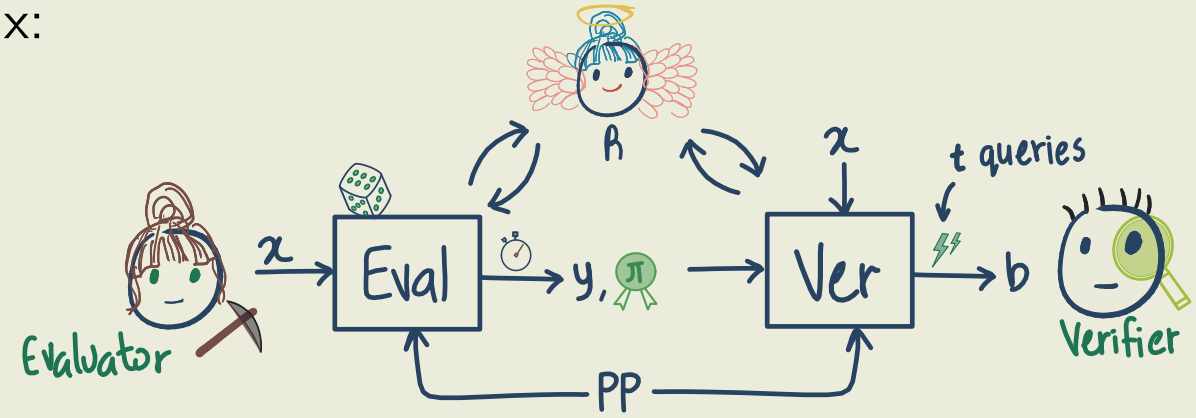

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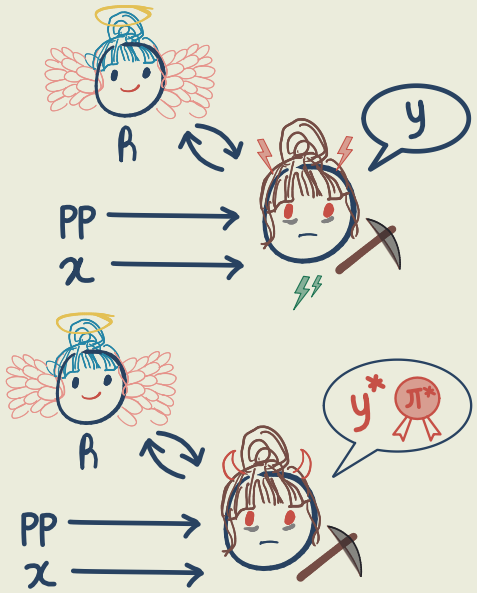

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How to Break Sequentiality?...

Assumptions: perfect sequentiality, public set-up and deterministic Eval

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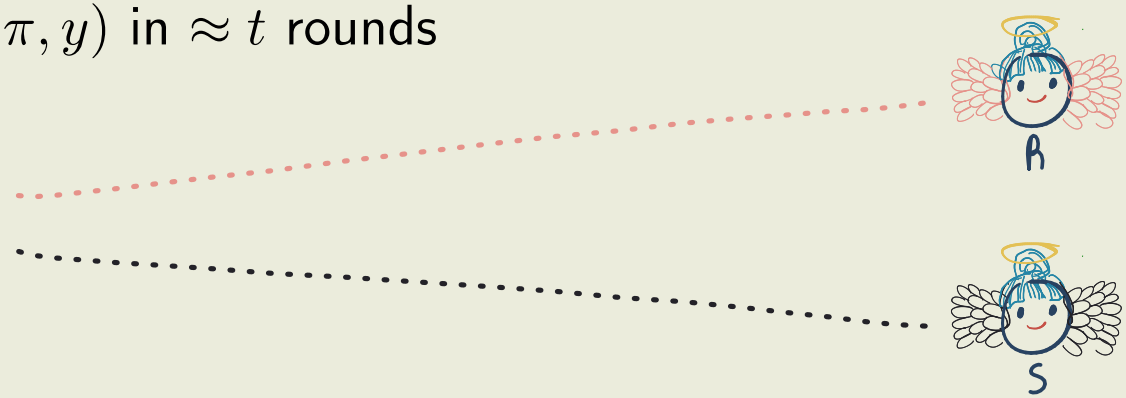



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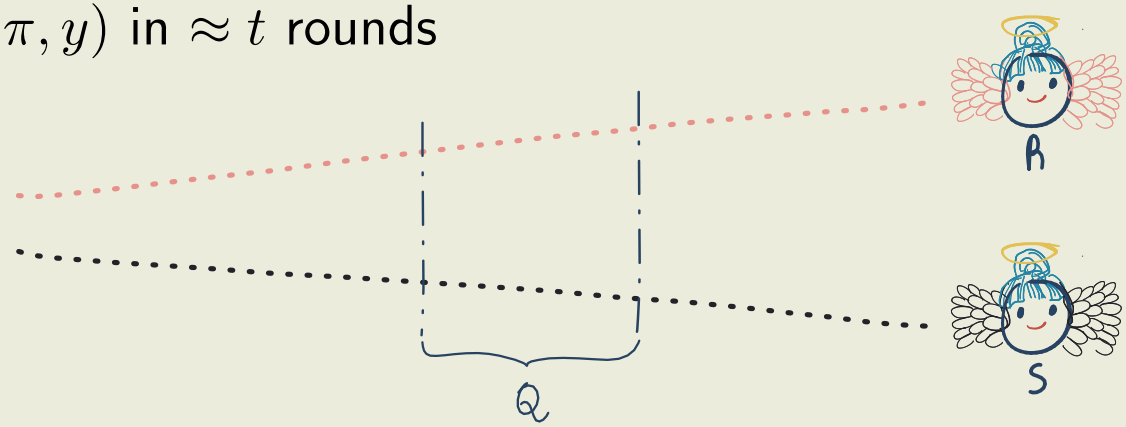

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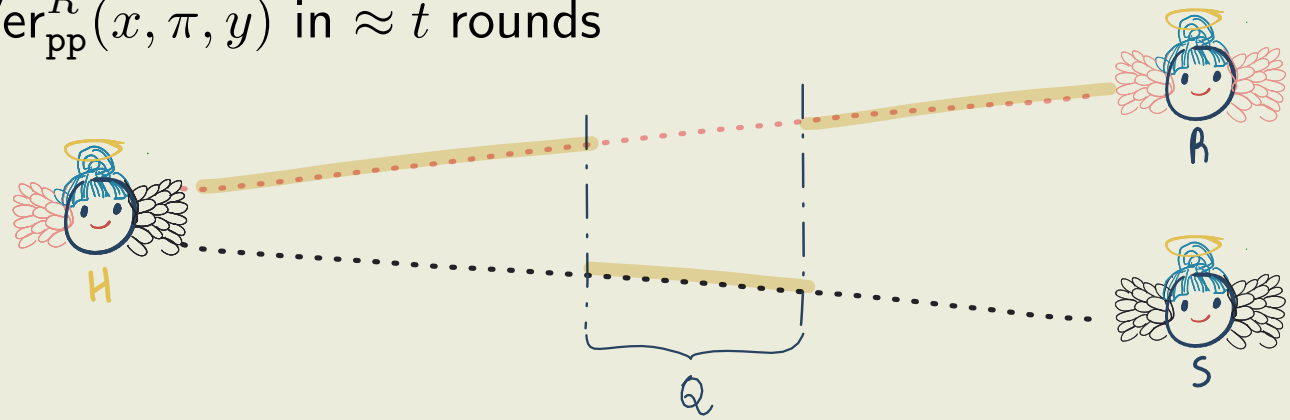

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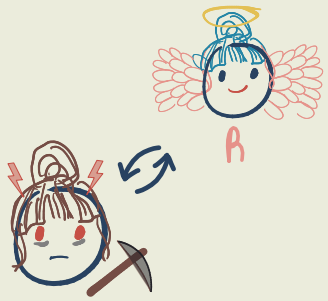

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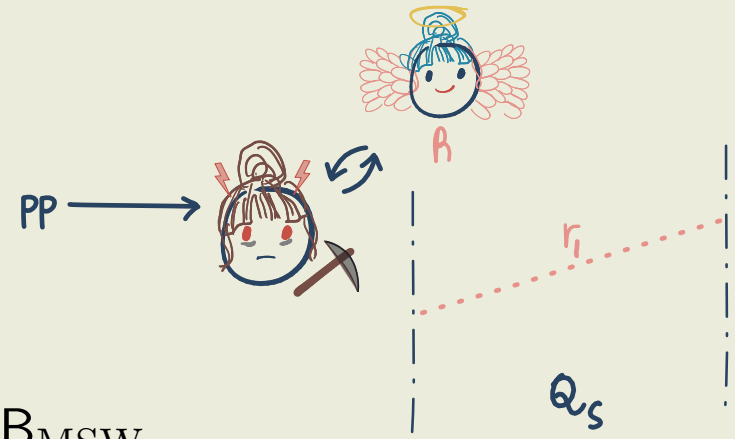

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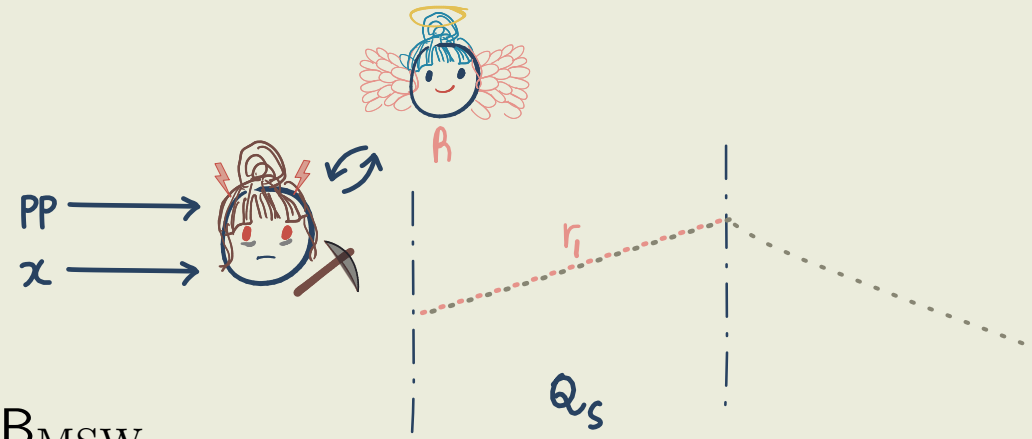

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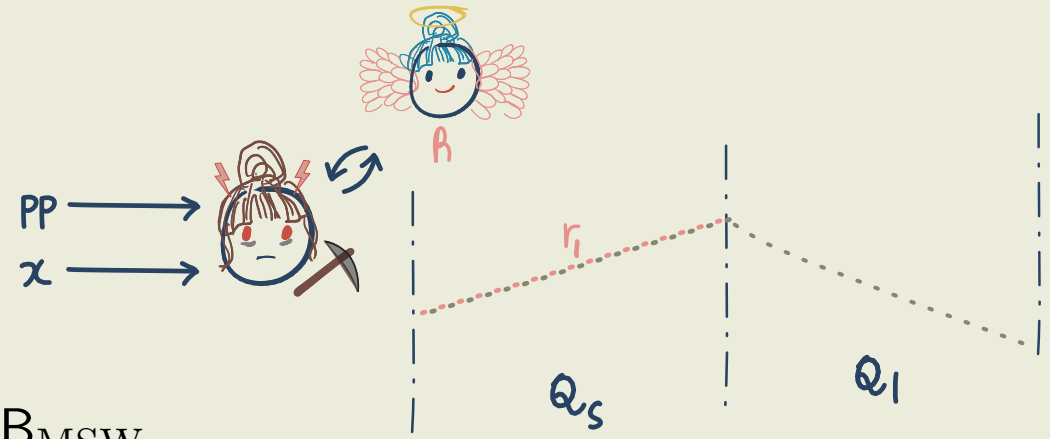

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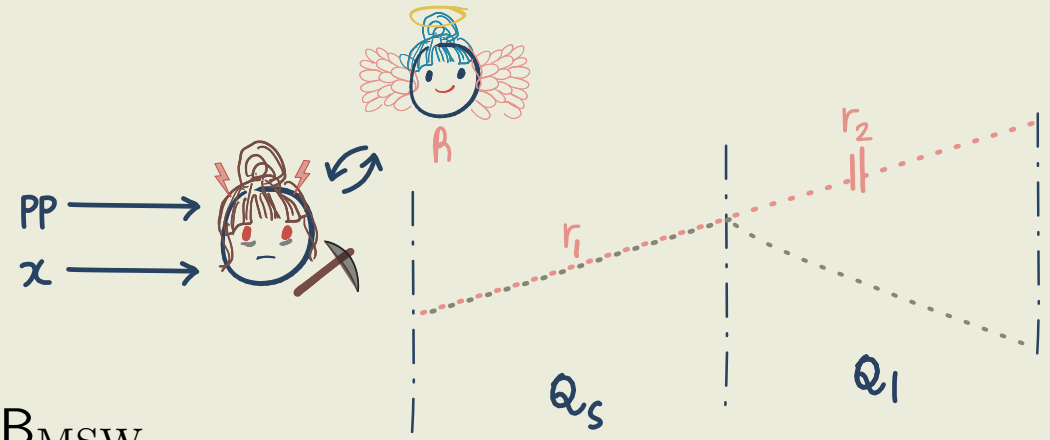

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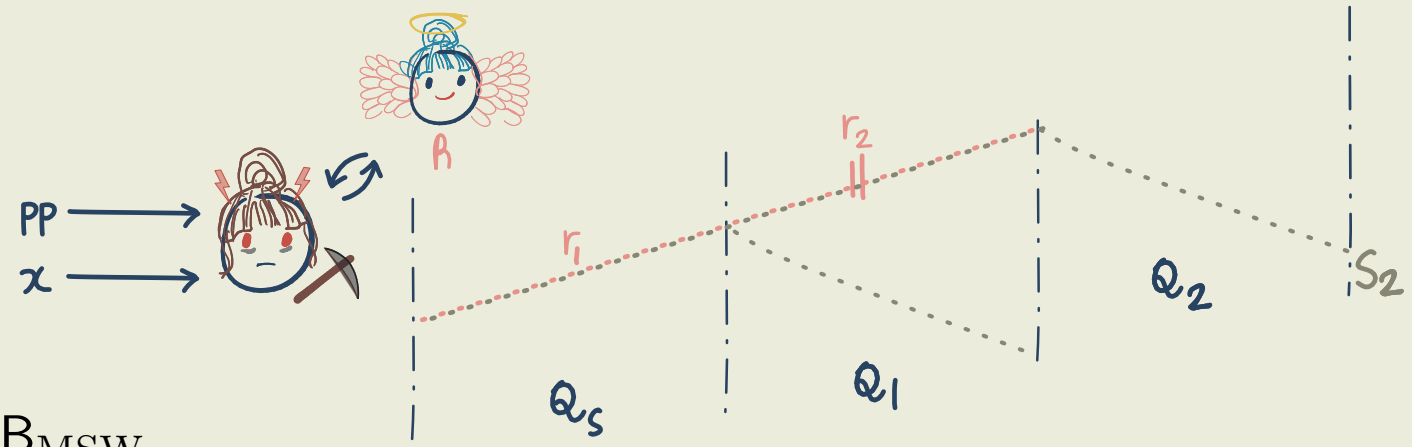

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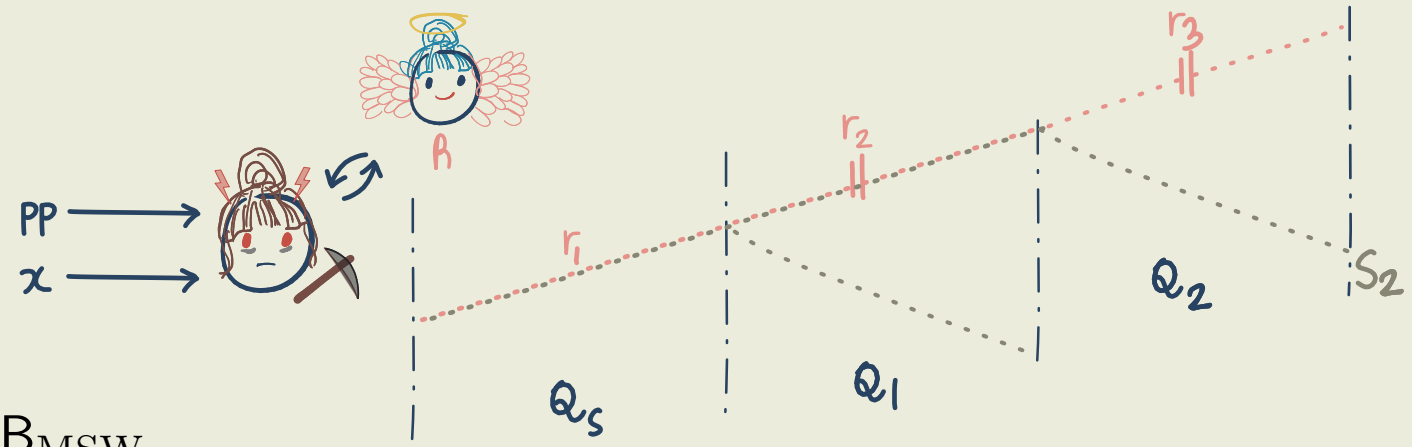

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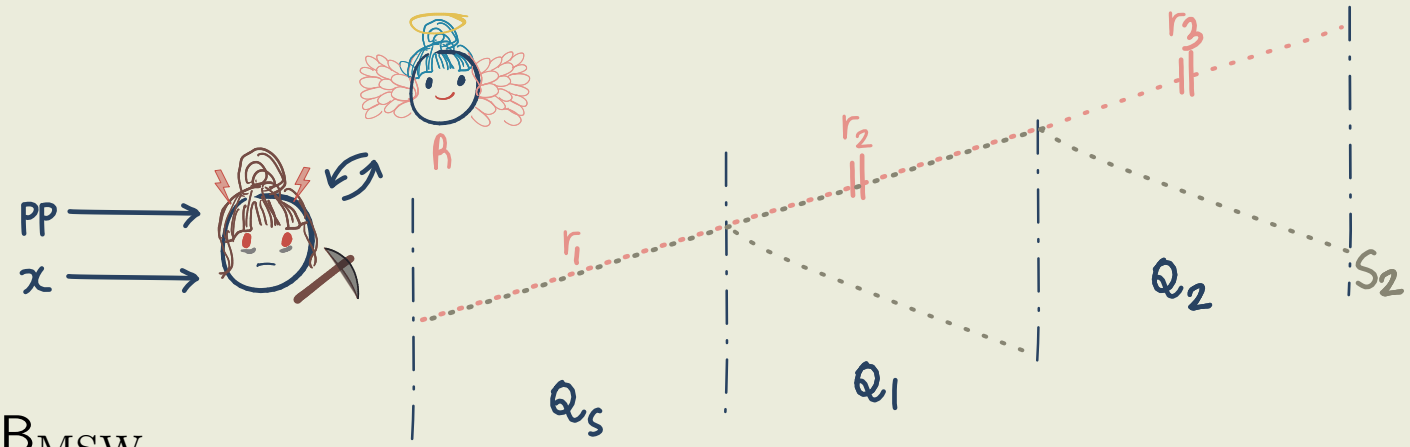

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7: Output y_{i^*} for $i^* \leftarrow [1, 2t]$

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Claim 1

$y_{i^*} = y$ holds with probability at least $1/2$

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Proof.

- $\mathrm{B}_{\mathrm{MSW}}$ "learns" in round i if $\mathcal{Q}_i$ *contains at least one query* from $\mathcal{Q}_V$

$\mathcal{R}$-queries by $\mathsf{Verify}^{\mathcal{R}}_{\mathsf{pp}}(x, \pi, y)$

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   - Since $|\mathcal{Q}_V| \leq t$ *and* R is queried on $\mathcal{Q}_i$ subsequently
2. Subclaim 2: If $\mathrm{B}_{\mathrm{MSW}}$ does not learn in round i^* then $y_{i^*} = y$
   - If $y_{i^*} \neq y$, then *uniqueness violated*

How to Break Sequentiality?...

Claim 1

$y_{i^*} = y$ holds with probability at least $1/2$

Proof.

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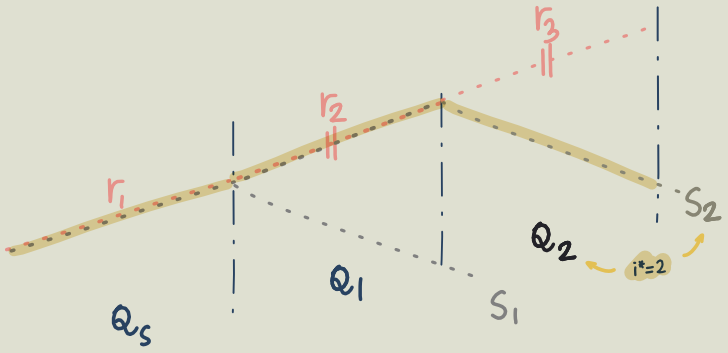

$$H(q) := \begin{cases} S_{i^*}(q) & \text{if } q \in \mathcal{Q}_{i^*} \\ R(q) & \text{otherwise} \end{cases}$$

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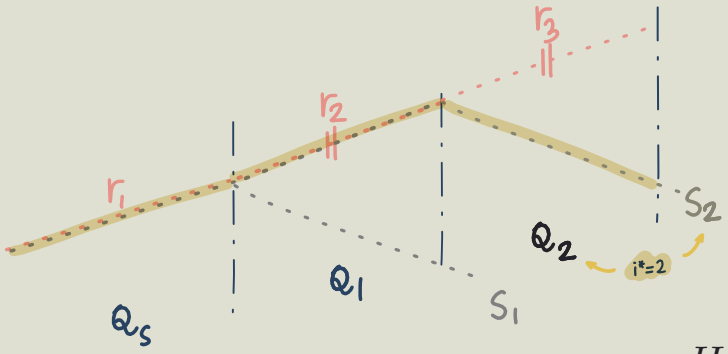

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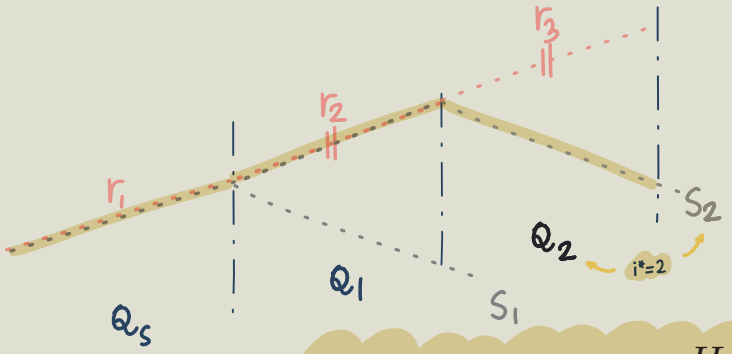

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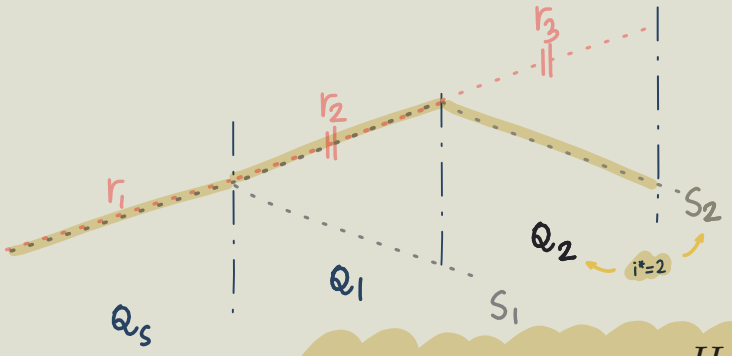

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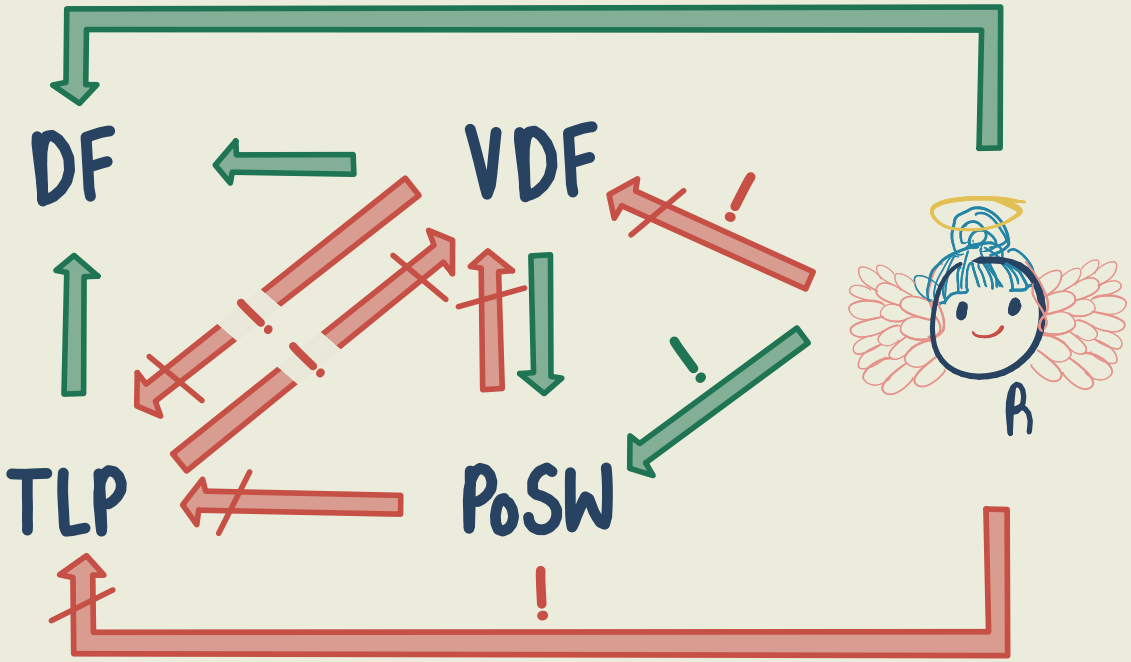
H consistent with S_{i^*} ↗ ↖ $\mathcal{Q}_{i^*} \cap \mathcal{Q}_V = \emptyset$

In Summary



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